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Internal Correction of Dead-reckoning Errors With the Smart Encoder Trailer

by
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ABSTRACT

This paper presents an innovative method for accurate mobile robot dead-reckoning, called ***I**nternal **P**osition **E**rror **C**orrection* (IPEC). In previous work, the IPEC method was successfully implemented on a specially designed mobile robot with two differential drive axles, called the *Multi-Degree-of-Freedom* (MDOF) mobile robot. Experimental results with the MDOF robot showed consistently **one to two orders of magnitude better dead-reckoning accuracy** than systems based on conventional dead-reckoning. Yet, the IPEC system requires neither external references (such as navigation beacons, artificial landmarks, known floorplans, or satellite signals), nor inertial navigation aids (such as accelerometers or gyros).

This paper focuses on our current efforts to implement the IPEC method on a device that can be added to any existing mobile robot. This device, called the "***S**mart **E**ncoder **T**railer*" (SET), is a small, single-axle trailer with an incremental encoder on each of its two wheels. Although the SET is not functional yet, recent simulation results combined with experimental results from the (similarly configured MDOF vehicle) strongly suggest the feasibility of the SET implementation.

1. Introduction

In most mobile robot applications two basic position-estimation methods are employed together: *absolute* and *relative* positioning [Borenstein and Koren, 1987; Hongo et al, 1987]. Relative positioning is usually based on dead-reckoning (i.e., monitoring the wheel revolutions to compute the offset from a known starting position). Dead-reckoning is simple, inexpensive, and easy to accomplish in real-time. The disadvantage of dead-reckoning is its unbounded accumulation of errors. Typical dead-reckoning errors will become so large that the robot's internal position estimate may be unacceptably wrong after as little as 10 m of travel [Gourley and Trivedi, 1994].

Absolute positioning methods usually rely on (a) navigation beacons, (b) active or passive landmarks, (c) map matching, or (d) satellite-based navigation signals. Each of these absolute positioning approaches can be implemented by a variety of methods and sensors. Yet, none of the currently existing systems is particularly elegant. Navigation beacons and landmarks usually require costly installations and maintenance, while map-matching methods are either very slow or inaccurate [Cox, 1991], or even unreliable [Congdon et al, 1993]. With any one of these measurements it is necessary that the work environment be either prepared or be known and mapped with great precision. Satellite-based navigation can be used only outdoors and has poor accuracy (on the order of several meters) when used in real-time, during motion.

Another approach to the position determination of mobile robots is based on inertial navigation with gyros and/or accelerometers. Our own experimental results with this approach, as well as the results published in a recent paper by Barshan and Durrant-Whyte [1993], indicate that this approach is not advantageous. Accelerometer data must be integrated twice to yield position, thereby making these sensors exceedingly sensitive to *drift*. Another problem is that accelerations under typical operating conditions can be very small, on the order of 0.01 g. Yet, fluctuation of this magnitude already occur if the sensor deviates from a perfectly horizontal position by only 0.5^{o}, for example when the vehicle drives over uneven floors. Gyros can be more accurate (and costly) but they provide information only on the rotation of a vehicle. A known problem with gyros is *drift*. Drift affects gyro-measurements especially because gyros measure rotational velocity, which then needs to be integrated to compute orientation.

In recent work we have developed a new method for accurate and reliable positioning with mobile robots, called ***I**nternal **P**osition **E**rror **C**orrection* (IPEC) [Borenstein, 1994, 1994V]. The IPEC method can provides order(s) of magnitude better accuracy than even the most accurate dead-reckoning vehicle, yet it requires neither external references (such as navigation beacons, artificial landmarks, known floorplans, or satellite signals), nor inertial navigation aids (such as accelerometers or gyros). Furthermore, the IPEC method corrects not only systematic (internal) errors, such as different wheel diameters, but also non-systematic (external) errors, such as those caused by floor roughness, bumps, or cracks in the floor.

In principal the IPEC method requires two sets of encoders mounted on two independent axles, for a total of four incremental encoders. The IPEC method also requires that the two axles are kinematically related to each other in such a way that the dead-reckoning information from one axle could be used to compute the position of the other. We tested the IPEC method initially with a rather unique mobile robot, called the *Multi-Degree-of-Freedom* (MDOF) vehicle, which was available at our lab and which happened to meet the kinematic requirements of the IPEC method (as can be seen in Fig. 1). With the MDOF vehicle implementation the IPEC method was demonstrated to provide consistently and repeatably **one to two orders of magnitude better positioning accuracy** than systems based on conventional dead-reckoning. However, the MDOF vehicle has a very unique kinematic design which is commercially feasible only in certain specialized applications.

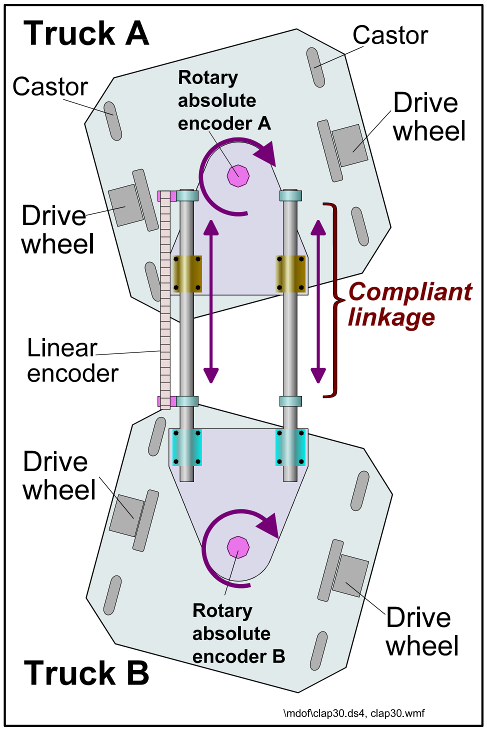


Figure 1:: The MDOF vehicle with compliant linkage was used in an earlier implementation of the IPEC method.

To make the IPEC method more widely applicable we have begun work on implementing the IPEC method on a device called the "***S**mart **E**ncoder **T**railer*" (SET). The SET is more generally applicable because it can be attached to most existing mobile robots without substantial design changes, even as a retrofit. It should be noted that the SET differs substantially from seemingly similar attachments like the one shown in Fig. 2. This trailer was designed to reduce dead-reckoning errors by providing carefully machined precision wheels that are more accurate in their measurements than the robot's drive wheels. While the trailer indeed improves dead-reckoning accuracy to some degree, it will still produce severe measurement errors when traveling over bumps, cracks, or other irregularities on the floor. In contrast, the SET uses the IPEC method to *actively detect and correct dead-reckoning errors*. With the IPEC method the SET corrects not only systematic errors (like a well-designed "conventional" encoder trailer could do), but it also corrects *non-systematic* dead-reckoning errors ,such as those caused by floor roughness, bumps, or cracks in the floor.

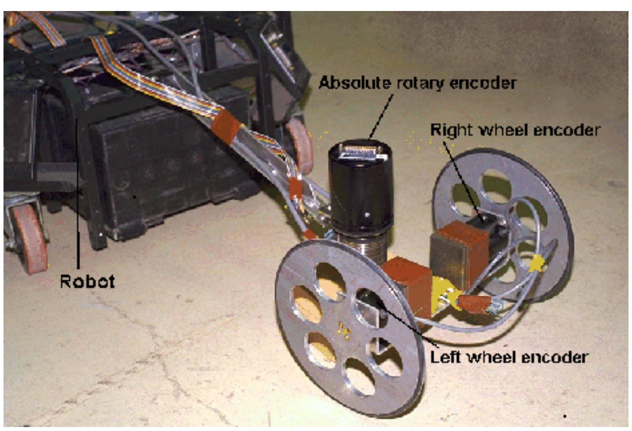


Figure 2:: This "conventional" encoder trailer was recently developed at the University of Michigan for use with tracked robots. The *smart encoder trailer* will look similar, but will have the absolute encoder located closer to the robot to optimize the implementation of the IPEC method.

The remainder of this paper is organized as follows: Section 2 reviews certain relevant properties of dead-reckoning errors. Section 3 explains the implementation of the IPEC method on the SET. Section 4 presents simulation results and Section 5 shows experimental results from the IPEC method implemented on the MDOF vehicle.

2. PROPERTIES OF DEAD-RECKONING ERRORS

In this section we discuss the characteristics of the two distinct types of dead-reckoning errors found in mobile robot navigation: non-systematic and systematic errors. Later in this section we will introduce the *Growth-Rate Concept* for non-systematic dead-reckoning errors. The IPEC method, makes use of this concept to detect and correct non-systematic errors.

2.1 Nonsystematic dead-reckoning errors

Most surfaces of typical concrete or asphalt floors are strewn with cracks, bumps, and sometimes debris, along with the inherent roughness of the floor surface. Such *irregularities* are always present in various degrees, depending on how well the work environment can be controlled. Even in well-controlled environments there is a chance for unexpected objects (e.g., an object dropped by a person or another mobile robot). Because of the random nature of such irregularities they cause *non-systematic errors*. Non-systematic dead-reckoning errors may also be caused by excessive wheel slippage, for example due to a fluid spill or a collision. These error sources can neither be avoided nor can they be compensated for with conventional dead-reckoning. To provide a physical sense for the effect of a typical non-systematic error we have listed path errors resulting from traversing a bump of 10 mm height in Table I.

Examining Table I, one should note that the resulting orientation error $\Delta\theta_a$ is the *most significant error* in the system [Feng et al, 1993], because it will cause an ***unbounded*** lateral error e_{lat}, which grows proportionally with distance at a rate of $e_{lat}(D) = D{\cdot}\Delta D/b = D \sin \Delta\theta_a$ (where D is the distance traveled since clearing the bump, and b is the wheelbase). As an example, Table I shows that the lateral error of a robot after 10 m travel (and after clearing the bump) would be $e_{lat}(D{=}10\text{ m}) = 77$ mm.

Table 1:
Sample path errors after traversing a bump

<u>Physical Dimensions</u>	
Wheelbase b	340 mm
Wheel radius R	75 mm
Height of bump h	10 mm
<u>Computed Results</u>	
Linear error ΔD	2.63 mm
Orientation error $\Delta\theta_a$ (see Fig. 3)	0.44°
Lateral error after 10m travel e_{lat}(D=10m)	77 mm

2.2 Systematic dead-reckoning errors

Systematic dead-reckoning errors are related to properties of the vehicle, that is, they are independent of the environment. The dominant systematic errors are [Borenstein and Koren, 1985, 1987; Banta, 1988]:

a) <u>Unequal wheel diameters</u>. Mobile robots use rubber tires to improve traction. These tires are difficult to manufacture to exactly the same diameter. Furthermore, rubber tires compress differently under asymmetric load distribution. Both effects can cause substantial dead-reckoning errors.

b) <u>Uncertainty about the wheelbase</u>. The wheelbase is defined as the distance between the contact points of the two drive wheels of a differential drive robot and the floor. The wheelbase must be known in order to compute the number of differential encoder pulses that correspond to a certain amount of rotation of the vehicle. Uncertainty in the effective wheelbase is caused by the fact that rubber tires contact the floor not in one point, but rather in a contact area. The uncertainty about the effective wheelbase is on the order of 1-5% in most robots.

In conventional mobile robots systematic errors can be reduced to some degree by careful mechanical design of the vehicle and by vehicle-specific calibration. However, systematic errors cannot be eliminated completely because they depend partially on changing factors such as load distribution. However, we will see in Section 2.2.4 that systematic errors are automatically corrected by the IPEC method, just like non-systematic errors.

3. IMPLEMENTATION OF THE IPEC METHOD

3.1 The Smart Encoder Trailer

The *Smart Encoder Trailer* (SET), shown in Fig. 3, is currently being built at the University of Michigan. In order to be fully functional, the system will comprise a towing vehicle, which will be called "robot," and the attached encoder trailer, which will be called "trailer." To simplify the following explanation, we will assume that the trailer is linked to the robot at the robot's center point, although linkage at the rear of the robot is also possible. An absolute rotary encoder is located at this joint to measure the relative angle between the robot and the extension link.

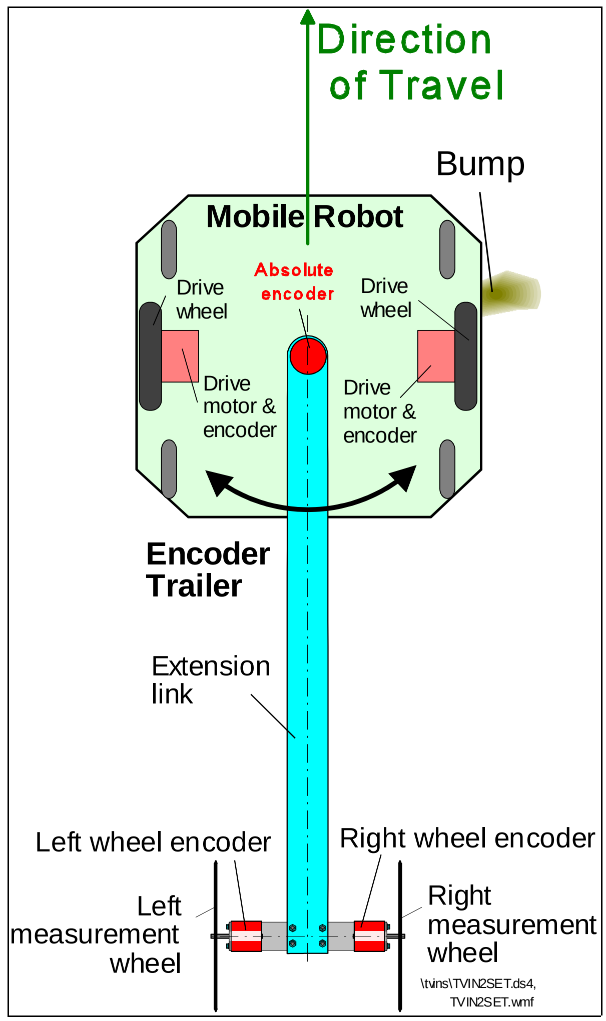


Figure 3: Robot and *Smart Encoder Trailer* are initially aligned (for the purpose of explanation) when the robot reaches a bump.

3.2 Internal Position Error Correction (IPEC)

The implementation of the IPEC method on the robot/trailer system of Fig. 3 is explained in Fig. 4. Figure 4 shows a line labeled "direction after traversing the bump," which is the (unintended) direction of the robot after it cleared the bump. Since this direction differs from the intended (straight ahead) direction as the result of a dead-reckoning error, the robot still "believes" it was traveling straight ahead. Consequently, the robot would expect the center of the trailer to be straight behind, along the dotted line labeled L_e in Fig. 4. Using dead-reckoning data from both the robot and the trailer, the robot can always compute this *expected direction* to the center of the trailer, whether both are traveling straight or along a curved path. This *expected* direction can then be compared to the *measured* direction, which is readily available from the absolute rotary encoder on the center of the robot. The difference between the *expected* direction and the measured direction is the *measured* orientation error $\Delta\theta_m$. $\Delta\theta_m$ can then be used (as will be explained below) to correct the computed orientation of the robot, which was based on dead-reckoning. The orientation error of the trailer can be determined in a similar way, relative to the center of the robot.

It is evident from Fig. 4 that the *measured* orientation error $\Delta\theta_m$ is not identical to the *actual* orientation error $\Delta\theta_a$, and one must ask how we can correct the original dead-reckoning orientation if we don't know the *actual* error. The answer to this question is that $\Delta\theta_m$ is *almost* identical to $\Delta\theta_a$, and that for the kinematic configuration of the robot/trailer system this *near-identity* is guaranteed under all operating conditions (with a few known exceptions). The *conceptual* proof for this claim is given in the *Growth Rate Concept* for dead-reckoning errors, which was first formulated in [Borenstein, 1994].

3.3 The *Growth-Rate Concept* for dead-reckoning errors

The *Growth-Rate Concept* is based on the insight that certain dead-reckoning errors develop quickly (*fast-growing errors)* while others develop slowly (*slow-growing errors*). For example, in a non-holonomous vehicle (such as a differential drive mobile robot or the encoder trailer of Fig. 3) one can safely assume that the vehicle or trailer do not move sideways under most normal operating conditions. This holds true even if the vehicle or trailer traverse bumps, cracks, or any other irregularity of the floor. Under these "normal" operating conditions, the only way a substantial lateral dead-reckoning error can (and will) develop is as a result of a preceding orientation error. In other words, when the vehicle or trailer traverse a bump on the ground, they will *immediately* experience a significant orientation error (thus, a *fast-growing error*), out of which a lateral position error will develop in subsequent travel (a *slow-growing error*). For example, the bump in Table I causes an orientation error of $\Delta\theta_a = 0.44^\circ$ (see Fig. 4). This error will be "fully developed" within one or two sampling intervals (assuming T_s = 40 ms) or a few centimeters of distance traveled. This orientation error can be measured by the absolute encoder on the robot and subsequently it can be corrected. In contrast, the *slow-growing* lateral position error at the end of one sampling interval is less than e_{lat} = 0.15 mm, as shown for the same numeric example in [Borenstein, 1994]. Borenstein [1994] further demonstrates that for a geometry similar to that of the robot/trailer system the lateral error (in the same example) reduces the accuracy of the orientation error measurement only by $\epsilon = 0.01^\circ$. These considerations hold true in the basic case in which only the robot encountered a bump while the trailer retained its heading. However, even if the trailer also encountered a bump *during the same sampling interval,* its lateral error would be similarly small. Neither this lateral error nor the orientation error of the trailer would cause a significant error in the orientation measurement of the robot relative to the

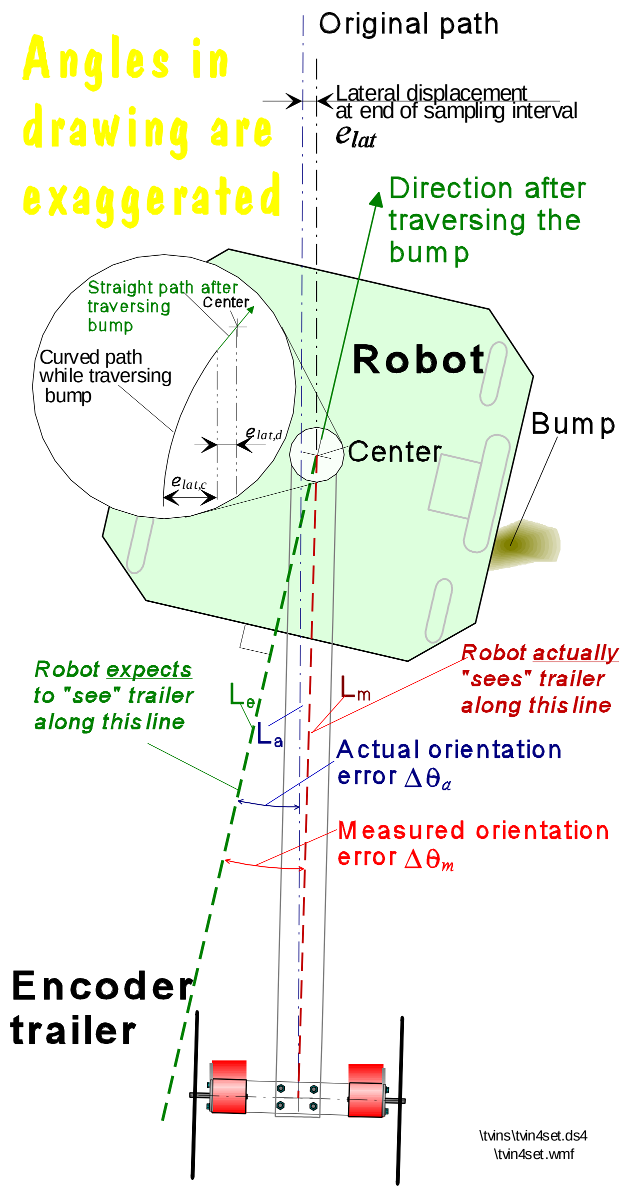


Figure 4: After traversing a bump, the resulting change of orientation of the robot can be measured relative to the trailer.

trailer or vice versa. Yet, even in the extreme and rare case of both robot and trailer encountering bumps during the same sampling interval, the inaccuracy of the orientation error measurement would be $\epsilon = 2\times0.01^{o} = 0.02^{o}$, which translates into $0.02/0.44\times100 = 4.5\%$ inaccuracy in measuring the orientation error of the robot relative to the trailer.

3.4 Additional Considerations

The IPEC method can detect only rotational errors, but not translational errors. However, rotational errors are much more severe than translational errors, because orientation errors cause the ***unbounded*** growth of lateral position errors. We can further distinguish two kinds of translational errors: *pure* and *composite*. *Pure* translational errors occur when both wheels traverse bumps of similar height during the same sampling interval. These errors cannot be detected with the IPEC method, but they are rare in practice and they produce only small and finite position errors. *Composite* translational errors occur when only one wheel traverses a bump, thereby causing a translational ***and*** a rotational error. Since we can detect the rotational error, we can also derive correction-terms for the translational portion of a composite error.

3.5 Systematic dead-reckoning errors

In Section 3.2 we explained the IPEC method with regard to non-systematic errors (e.g., a bump). Another source of dead-reckoning errors is known as *systematic errors*. Systematic errors are usually caused by imperfections in the design and mechanical implementation of a mobile robot. In conventional, differential-drive mobile robots the two most notorious systematic errors are caused by different wheel diameters and the uncertainty about the effective wheelbase [Borenstein and Koren, 1985, 1987; Banta 1988].

Systematic errors are particularly grave, because they accumulate constantly. On most smooth indoor surfaces systematic errors contribute much more to dead-reckoning errors than non-systematic errors. However, on rough surfaces with significant irregularities, non-systematic errors are dominant. One hard-to-defuse criticism of work aimed at reducing systematic dead-reckoning errors *alone* is the claim that any *unexpected* irregularity can introduce a huge error, no matter how effective the reduction of systematic errors was.

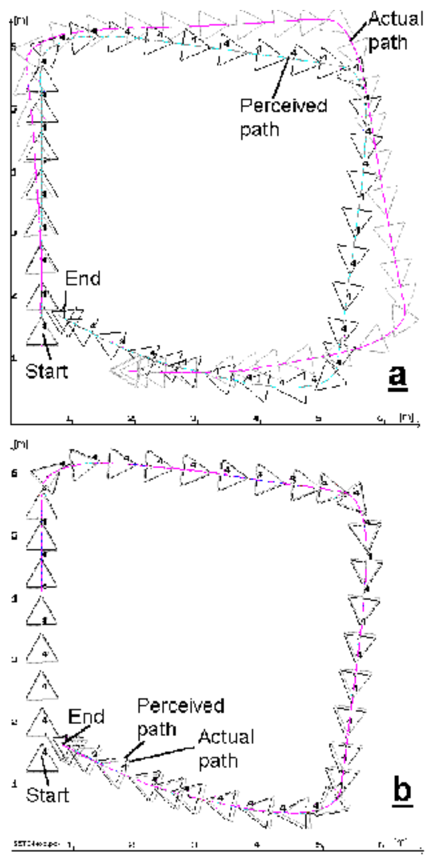


Figure 5:: Robot trajectories of simulation runs with frequent disturbances. The double images show the difference between "actual" and "perceived" positions of the robot, due to dead-reckoning errors. (a) **IPEC disabled**, and (b) **IPEC enabled**.

One important advantage of the IPEC method is its ability to correct both *non-systematic* errors **and** *systematic* errors, provided the systematic error causes a dead-reckoning error in orientation. For example, consider a mobile robot programmed to move straight ahead. Unequal wheel-diameters will cause the mobile robot to follow a curved path, instead. In the robot/trailer system equipped with IPEC, the rotation of the robot following a curved path (due to unequal wheel diameters) can be detected: The IPEC method will trigger a correction as soon as the accumulated orientation error of the robot exceeds the resolution of the absolute encoder.

4. SIMULATION RESULTS

The functioning of the *Smart Encoder Trailer* (SET) with IPEC was tested in a simulation program. In this simulation it was assumed that the trailer was joined to the robot at the rear of the robot at an offset of 300 mm from the center of the robot. The length of the Extender Link was 500 mm. Figure 5a shows a simulation run without IPEC while Fig. 5b shows the same run with IPEC enabled. In both figures the approximately square-shaped trajectory of the robot is shown as a continuous line, with "snapshots" of the robot symbolized by a triangle. The total path was approximately 18 m long. The program simulated 10 mm high disturbances at 0.5 m intervals. Every disturbance is marked by a '4' in Fig. 5, and the disturbance always occurred under the left wheel of the robot (to avoid mutual cancellation of subsequent errors).

In the run without IPEC (Fig. 5a) one can clearly distinguish between two traces of the robot. One trace shows the *actual* trajectory of the robot, while the other trace shows the "*perceived*" trajectory, i.e., where the robot "thought" it was, based on dead-reckoning. After completing the 18 m long path, the "*actual*" and the "*perceived*" positions differed by 110 and 90 cm respectively in x and y direction, and the orientation error was 26.0^{o}.

By contrast, the same run with the same disturbances but with IPEC *enabled* yielded a final position error of 4.2 and 4.8 cm, respectively, and an orientation error of 1.0^{o}. These simulation results correspond well to actual experimental results obtained with the MDOF vehicle, as shown in the following section.

5. EXPERIMENTAL RESULTS WITH THE MDOF VEHICLE

In this section we report on preliminary experimental results that prove the validity of the IPEC method. Since a prototype of the proposed SET does not exist yet, the preliminary experiments were performed on an experimental vehicle with characteristics that are somewhat similar to those of the robot/SET system. This experimental vehicle is the *Multi-Degree-of-Freedom* (MDOF) vehicle with *compliant linkage*, which was developed at our lab in earlier work [Borenstein, 1993].

All experiments with the IPEC method implemented on the MDOF vehicle were conducted on fairly smooth concrete floors. Controlled irregularities were generated by repeatedly placing a piece of 10 mm diameter cable under one side of the vehicle. We will refer to these irregularities as "bumps." All experiments started and ended near an L-shaped reference corner. Three ultrasonic sensors were mounted on the MDOF vehicle, two sensors were facing the long side of the L-shaped corner, the third sensor faced the short side. The ultrasonic sensor system allowed measurement of the absolute position of the vehicle to within ±2 millimeters in the x and y directions, and to about ±0.25° in orientation.

At the beginning of each run a sonar measurement was taken to determine the starting position of the vehicle. The vehicle then traveled through a pre-programmed 7×4 m rectangular path with smooth 90° turns at the corners and a total travel length of approximately 24 m (see Fig. 6). To provide fluid, uninterrupted motion, the programmed path did not require the vehicle to stop at the intermediate points — passing-by at a distance of less than 0.2 m was sufficient.

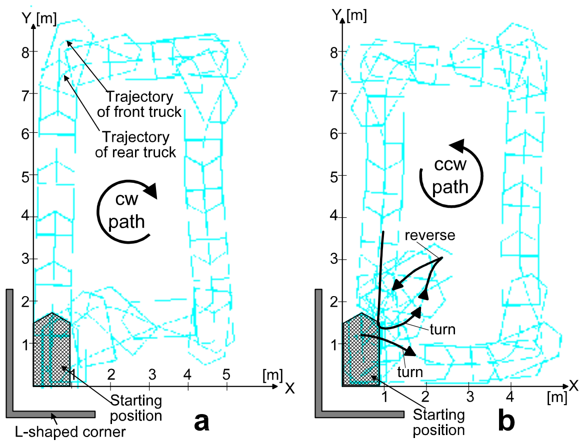
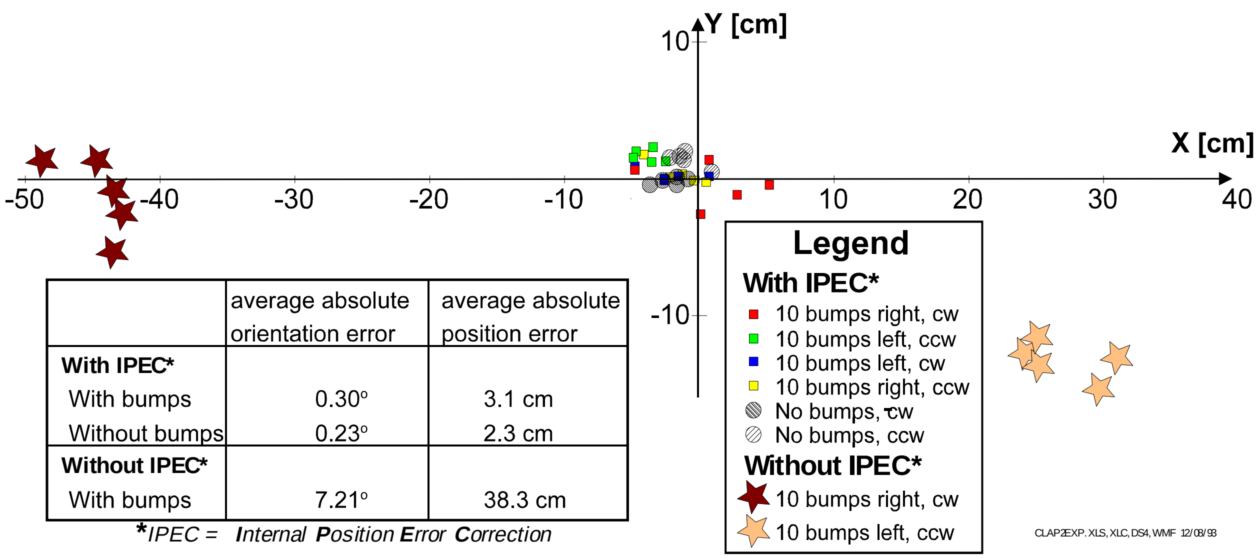


Figure 6: The *Rectangular Path Experiment* was performed in clockwise (cw) and counter-clockwise (ccw) direction. Some sideways and backward maneuvering was necessary to return to the home position.

After returning to the L-shaped corner, the *perceived* position (i.e., the position the vehicle "thought" it had, based on dead-reckoning) was recorded. Then, a sonar measurement was taken to determine the *absolute* position. The difference between the absolute position and the perceived position is called the *return position error*. The average speed in all runs was slightly below 0.5 m/sec.

Figure 7 shows the *return position errors* under different test conditions. The vehicle ran through the path for 10 runs in cw, and 10 runs in ccw direction. In each of these runs the vehicle had to traverse 10 bumps. In one half of the runs bumps were located under the right-side wheels of both trucks, and in the other half of the runs under the left-side. The return position errors of these runs are marked by small squares (see *Legend* in Fig. 7). None of the 20 runs produced an error of more than 5 cm. Also shown in Fig. 7 are the results of five cw and five ccw runs with IPEC but without bumps (marked by small circles). Note



	average absolute orientation error	average absolute position error
With IPEC*		
With bumps	0.30°	3.1 cm
Without bumps	0.23°	2.3 cm
Without IPEC*		
With bumps	7.21°	38.3 cm

IPEC =** *Internal* ***Position Error Correction

Figure 7: Return position errors after completing the *Rectangular Path Experiment.* Total travel distance in each run: 21m.

that the results with bumps are almost indistinguishable from the results without bumps. In a further experiment the vehicle ran through the path with bumps, while the IPEC function was *disabled* (i.e., using normal dead-reckoning like conventional mobile robots). The results of these runs are marked by stars in Fig. 7. Also noted in the inset table in Fig. 7 are the resulting *average absolute orientation errors* of these runs, defined as

$$\epsilon_{\theta,avrg} = \frac{1}{n}\sum_{i=1}^{n} |\epsilon_{\theta,i}|$$

One might recall that for longer distances the orientation errors cause the lateral position errors to grow without bound. The results in Fig. 7 show that the IPEC method resulted in a more than 20-fold reduction in orientation errors. Indeed, in longer paths with more disturbances one should expect even better improvements, because the average absolute orientation error of $\epsilon_{\theta,avrg} = 0.3^o$ (obtained with IPEC) is just about equal to the accuracy with which the *actual* position of the MDOF could be measured with the three onboard ultrasonic sensors.

6. CONCLUSIONS

This paper presented a new method for accurate mobile robot dead-reckoning, called *Internal Position Error Correction* (IPEC) and the implementation of that method on a device called *Smart Encoder Trailer* (SET). Simulation results with the SET showed an improvement of 1-2 orders of magnitude in dead-reckoning accuracy as compared to conventional mobile robots. The SET promises to be of practical value in many mobile robot applications, since almost all mobile robots use dead-reckoning. The SET is particularly practical because it can be added to existing mobile robots, with only minimal modifications to the original design of the vehicle. Furthermore, the components needed to build an SET are all commercially available; an SET can be build for less than $1,000.

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