Virtual Vehicle: parcel manipulation and dynamics with a distributed actuator array

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ABSTRACT

A novel materials handling system is being developed at Carnegie Mellon University's Mechanical Engineering Department. This systems contains an array of cells, each of which has two actuators. The two actuators are orthogonally oriented motorized roller wheels which, in combination, can generate a vector of motion in any planar direction. This work develops control laws for transporting and manipulating objects which rest on the array. Towards this goal, we consider the dynamics of parcel transport and manipulation. The parcel dynamics are based on an exact discrete representation of the system, unlike other methods where a continuity assumption is made. Two types of contact models are considered. This work extends the previous one-dimensional discrete model into two-dimensions.

Keywords: Distributed Actuation, Materials Handling

1. INTRODUCTION

Many applications require objects to be transported and manipulated in the plane. Two such applications are flexible manufacturing and package handling. Flexible manufacturing requires that working part paths be arbitrarily specified and that these paths be readily updated by a "factory programmer." Package handling systems, such as airport baggage handling, require parcels to be sorted, oriented, and transported to one of several locations.

Two existing methods of material transfer are conveyor belts and robotic manipulators. The conveyor belts are able to transfer many large heavy objects for long distances, but lack the ability to orient precisely and to re-order the objects. Furthermore, while conveyor belts allow objects to be diverted to different destinations, they are not well suited to merging objects from different sources. Robots, on the other hand, are able to precisely position and orient objects, but only one at a time, and are limited by strength and reach. The Virtual Vehicle system, described below, combines the benefits of conveyor and robotics transport systems while overcoming their limitations.

We are developing an alternative method for manipulating objects in the plane, where many small manipulation cells fixed in a planar array cooperate to handle objects. In our system, each cell consists of a pair of actuators whose combined action can effect force in any planar direction to a parcel resting on top of the array. Furthermore, each cell contains up to five binary sensors which detect the presence of an object. See Figure 1.

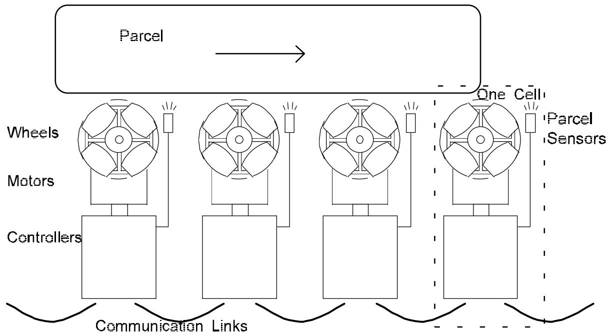

Figure 1. A few manipulator cells carrying a parcel.

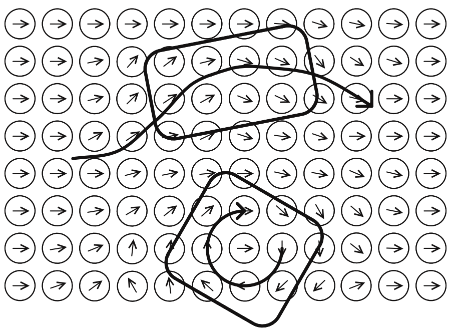

Figure 2. The Virtual Vehicle: Several parcels can be translated and rotated, independently.

In our system, the parcels are significantly larger than each cell; several cells handle a single object. Through proper coordination, parcels which ride on top of the array can be made to translate and rotate in the plane. Since sensing and actuation are distributed, each of many parcels can be manipulated independently, appearing as if each parcel were carried by a separate vehicle (Figure 2). Hence, the name *Virtual Vehicle.*

For many applications, a dedicated robot or conveyor is the simplest and most appropriate solution. There are cases, however, where features, such as additional flexibility and reconfigurability, are required. In these cases, the Virtual Vehicle possesses many advantages, including

- **Flexibility.** Since the actuation is distributed, multiple parcels can be manipulated independently. This allows for parcels to be sorted, re-ordered, and re-directed quickly. Objects of many sizes and shapes can be passed along easily, and multiple object pathways can be invoked in parallel.
- **Redundancy.** Since the Virtual Vehicle System is a massively parallel array of cells, if one cell breaks, the neighboring cells work around the broken cell by either diverting parcels around the broken cell or simply passing parcels over the it.
- **Modularity/Reconfigurability.** Many cells can be produced at a low cost because of their small size and relative simplicity. The cells are designed to "snap" together to form an array. This modularity allows the cells to be arranged in any configuration. The array can be easily reconfigured by moving cells and adding new cells. The modularity also enables easy repair because a broken cell is simply replaced.
- **Scalability.** Cells can be designed to carry objects of all sizes. For example, micromachined actuators can carry near-microscopic objects such as integrated circuit components, whereas small plastic wheels can carry suitcases (of many sizes) through airports and large truck tires can carry box cars around a ship/train yard.

The Virtual Vehicle can be used in conjunction with traditional robots and conveyor belts to form hybrid systems. For example, in airport baggage handling, long conveyors can be used to transport parcels over long distances while Virtual Vehicle arrays can be installed at conveyor junctions to sort and to re-direct parcel traffic. In flexible manufacturing, the Virtual Vehicle can be used to transport objects between robot workspaces where simple robots are used for object fixturing.

2. SYSTEM DESCRIPTION

2.1. Mechanical Configuration.

We have built a prototype system consisting of a small array of cells capable of transporting objects about the size of a bread box. Each cell consists of a pair of orthogonally oriented motorized roller wheels (Figures 3 and 4) which are capable of producing a force perpendicular to their axes, while allowing free motion parallel to their axes. Each wheel is driven through a gear reduction by a small DC motor.

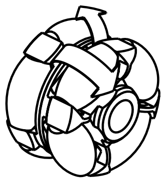

Figure 3. Roller wheel.

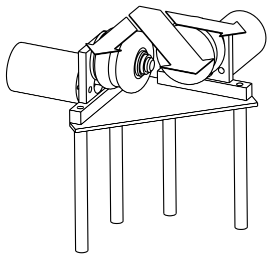

Figure 4. Prototype cell.

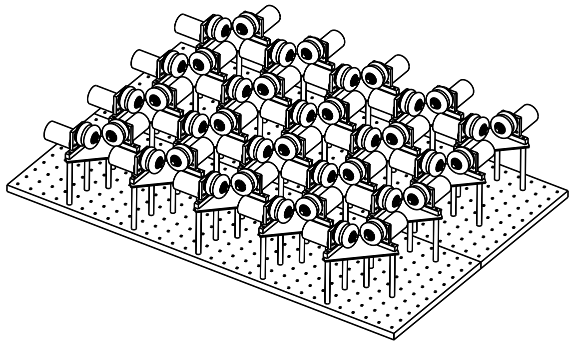

Figure 5. Two-dimensional array of cells.

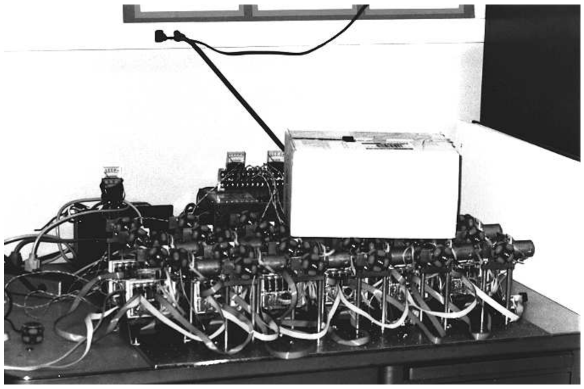

Figure 6. Current experimental setup.

Each of these cells is connected to a large breadboard style base (Figure 5) to create a regular array of manipulators. Currently, we have only 18 cells which can be arranged either in a single 1-D or a 2-D grid. A photograph of the experimental setup is shown in Figure 6.

2.2. Electronic Configuration

We are exploring the use of distributed control on the Virtual Vehicle because it is a relatively underdeveloped and interesting control paradigm, and because we believe that it would become impractical or impossible for a single computer to control hundreds or possibly thousands of cells. The controller of each cell is a single-board computer based on the Motorola MC68HC11 microprocessor. This is a fairly capable processor, and in an actual application, either multiple cells would be controlled by one processor, or a simpler processor would be used for better cost-effectiveness. The speed of the wheels is measured by a quadrature encoder. The presence of a parcel is sensed by a phototransistor which detects the shadow from the overhead lights. The motor is driven by pulse width modulation using an H-bridge circuit.

2.3. Network Configuration

An important aspect of our control scheme is the communication between cells. Each controller has an RS232 serial communications port. This port is multiplexed between each of the four neighboring cells, requiring a cell to actively select a neighbor in order to send or receive a message. Handshaking lines are necessary to alert a neighboring cell that a message is pending. A cell's processor polls through all four neighbor's handshaking lines to look for pending incoming messages.

The above described communication may seem awkward compared to standard local area network schemes where each cell would have an address, and all cells share a data bus. We require many communications to occur simultaneously on the array, which cannot be accomplished with a shared data bus. Also, we would like our network not to be limited by a maximum address space, or to require any cell to know its absolute location. Each cell need only be aware of its immediate surroundings. If any information needs to be sent further than one cell away, a message can be passed along the array.

3. PRIOR WORK

The concept of cooperative motion control has been examined in the past (see Mataric[1] for an overview). Reynolds[2] has emulated the flocking motion of birds using simple rules between independent agents which react to their surroundings. The idea of cellular robotics described by Wang and Beni[3] is similar to the Virtual Vehicle system in that a fixed array of robots cooperates to manipulate parts for manufacturing, but in cellular robotics, the manipulated parts are handled by only one robot at a time. Unlike the Virtual Vehicle, the parts are smaller than the robots. Much of the other work in this area has been concerning the cooperation of multiple mobile robots such as a robotic ant colony.[4,5] Such systems do not have a fixed spatial arrangement in contrast to the Virtual Vehicle.

Mason and Erdmann[6,7] provided a radical alternative to standard robotic manipulation by significantly simplifying the robot manipulator and developing manipulation algorithms for these low-degree-of-freedom sensorless mechanisms. Goldberg[8] developed an algorithm which orients to symmetry a part with a sequence of gripper open, close, and rotation operations without sensor information. This sequence of operations is termed a *squeeze.*

Böhringer, Donald, et. al.[9] applied this type of sensorless manipulation to an array of micromechanical actuators which were used to manipulate very small objects. Whereas Goldberg used a single centralized manipulator to orient objects, Böhringer used distributed actuation, but in both methods the control is centralized. The Böhringer system differs from the proposed system in that (i) it is on a small scale, (ii) the control is centralized, and (iii) the manipulation is entirely sensorless. It is worth noting that although their work has motivated some of the Principal Investigators' research, their work uses quasi-static analysis, and ignores dynamics because of the small scale application.

Recently, Kavraki[10] supplied further analysis of microactuated systems using elliptical potential fields to orient an object to symmetry without sensors. Potential functions are used to predict stable configurations of objects. Continuous potential functions adequately approximate Kavraki's and Böhringer's systems, because many micro-sized cells support a larger object. Potential field theory cannot be used for motion planning on the proposed system, because continuous functions do not adequately model the Virtual Vehicle. Only a small number of cells support an object. The Virtual Vehicle system is highly discrete whereas the systems considered by Kavraki and Böhringer are essentially continuous.

The Virtual Vehicle concept was introduced by the authors[11] where a continuous medium model of the system was used. This idea says that a regularly distributed array of interacting dynamic systems can be treated as a continuous medium, rather than a collection of discrete elements. This was demonstrated by assuming a simple control law for each cell in a row of cells based on the speeds of the neighboring cells. The equation of motion for each cell contains a finite difference term which is recognized as the approximation of a second partial derivative with respect to distance along the array. The resulting equation is equivalent to the transient heat conduction equation, with actuator velocity analogous to temperature. Physical analogies such as these are useful to understand the dynamics and to design behavior for such distributed systems. Simulations were performed to demonstrate these ideas.

More recently, prototype hardware was built and tested.[12] Simple tests, such as passing a cardboard box back and forth along a row of cells were performed. A distributed network and a communication language were developed which enables the cells to communicate effectively with each other. The network is briefly described in Section 2. The language involves the construction of short "sentences" which are made up of labeled pieces of data, or words, punctuated by a checksum. Such a sentence may be translated, for example, as "I know a parcel which is three cells away and which should go at a speed of 20."

In a previous paper by the authors[13] the first step was taken in examining the dynamics of a parcel carried by the Virtual Vehicle, where the one dimensional motion of the parcel along the array of cells was considered. In that paper, the forces between each cell and the parcel were calculated, and both a coulomb and a viscous-like friction law were considered. A particular case which was studied was where all the wheels in a row of cells spin in one direction, except for the cell on the end which spins in the reverse direction. Using both friction laws, the resulting parcel dynamics are that of a simple or damped harmonic oscillator, where the frequency, center of oscillation, and damping constant are constants which change as the parcel shifts from one set of supports to another. This reduction to a simple oscillator was also observed in the prototype system.

This paper extends our previous work into two dimensions with translaton and rotation. The ultimate goal of this work is to be able to predict and control the motion of one or more parcels with two-dimensional translation with rotation in the plane.

4. DYNAMICS OF MANIPULATION

Towards the goal of representing the dynamics of handling a parcel in the plane with an array of actuators, we will consider the dynamics of an array of cells transporting and rotating a parcel in the plane while it rests on a single arbitrary set of cells. Each of the cells is assumed to be spinning at a constant speed, and the horizontal force applied to the parcel is due to sliding friction. The horizontal force provided by a cell is a function of the weight supported by that cell.

In order to find the horizontal translation and rotation dynamics of the parcel, there are several steps to follow:

- Determine normal forces, $\overline{N}$, for a set of n supporting cells.
- Determine normal forces when only m cells out of the entire set of n cells support the parcel by masking off only the cells under the parcel.
- Apply a friction law to determine the horizontal forces and sum up forces and moments applied to the parcel.

4.1. Normal Forces

In order to solve for the supporting forces, we need to consider the equilibrium of the parcel in both the vertical (z) direction and in rotation about the x and y axes. Since equilibrium only provides three equations, and we have n supports, the system is statically indeterminant, and flexibility in the supports needs to be considered to solve for the remaining $n-3$ forces. These $n-3$ equations will be referred to as *compatibility* equations. The n equations will be represented in matrix form, and the vector of n normal forces, $\overline{N}$ can be solved for.

4.1.1. Parcel equilibrium

n cells arranged arbitrarily in the x-y plane have coordinates:

$$\overline{\mathbf{X}} = \begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ y_1 & y_2 & y_3 & \cdots & y_n \end{bmatrix} \tag{1}$$

A parcel, whose center of mass is located at $\overline{X}_c = \begin{bmatrix} x_c & y_c \end{bmatrix}^{\mathrm{T}}$ resting on some of these cells is supported by vertical normal forces:

$$\overline{N} = \begin{bmatrix} N_1 & N_2 & N_3 & \dots & N_n \end{bmatrix}^{\mathrm{T}} \tag{2}$$

Assume, for now, that all cells lie underneath the parcel. Any cell which does not lie under the parcel have $N_i = 0$. The parcel must be in equilibrium in the vertical direction and in rotation about the x and y axes. The vertical equilibrium of the parcel whose weight is W requires that

$$\sum_{i=1}^{n} N_i = W = [\ 1 \quad 1 \quad 1 \quad \cdots \quad 1\ ]\ \overline{N} \tag{3}$$

Rotation equilibrium about the x and y axes require the moments induced by the normal forces about any point (in this case, the origin of our coordinate system) sum to the moment about this point induced by the weight of the parcel. The moment equilibrium about the x and y axes, respectively, requires that

$$\sum_{i=1}^{n} N_i y_i = W y_c = \overline{y}\ \overline{N} \tag{4}$$

$$\sum_{i=1}^{n} N_i x_i = W x_c = \overline{x}\ \overline{N} \tag{5}$$

This, so far, yields three equations (3,4, and 5) and n unknowns (the n elements of $\overline{N}$).

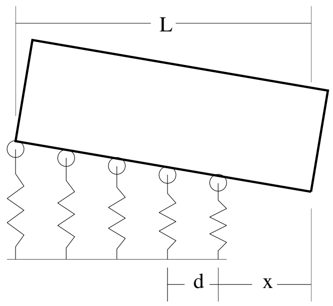

Figure 7. Flexible supports in many-cell case.

4.1.2. Compatibility

To solve for the remaining $n-3$ forces, we have to take into account flexibility in the system. A simple model of this flexibility is shown in side view in Figure 7. Each support is assumed to be a spring, with Hooke's Law ($N_i = K_s \Delta z_i$) representing the compression of the i^{th} cells under a normal load. Physically, this flexibility can be thought of as a flexible suspension under each wheel, as will be included in the next generation of our hardware array. In the current system, the flexibility is apparent in the bottom of the parcel, particularly when a rubbery material is used under the parcel to increase friction.

Assuming the bottom of the box is nominally flat, the flexibile cells conform to the bottom of the box to distribute the weight. All the cells (under the parcel) lie in this plane. If the plane of the bottom of the parcel is represented as:

$$z = a'x + b'y + c' \tag{6}$$

then each cell will obey the following displacement relation

$$\begin{aligned} \Delta z_i = K_s N_i &= a'x_i + b'y_i + c' \\ N_i &= ax_i + by_i + c \end{aligned} \tag{7}$$

4.1.3. Combine Equations

Equations 3, 4, and 5 along with n instances of Equation 7 supply $n+3$ equations and $n+3$ unknowns (n N_i's and 3 plane parameters, a, b, and c). This system of equations can be written in matrix form as follows:

$$\underbrace{\left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & \cdots & 0 & -1 & -y_1 & -x_1 \\ 0 & 1 & 0 & & 0 & -1 & -y_2 & -x_2 \\ 0 & 0 & 1 & & 0 & -1 & -y_3 & -x_3 \\ \vdots & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 & -y_n & -x_n \\ \hline -1 & -1 & -1 & \cdots & -1 & 0 & 0 & 0 \\ -y_1 & -y_2 & -y_3 & \cdots & -y_n & 0 & 0 & 0 \\ -x_1 & -x_2 & -x_3 & \cdots & -x_n & 0 & 0 & 0 \end{array}\right]}_{\overline{\mathbf{A}}} \underbrace{\left[\begin{array}{c} N_1 \\ N_2 \\ N_3 \\ \vdots \\ N_n \\ \hline c \\ b \\ a \end{array}\right]}_{\hat{N}} = \underbrace{\left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \hline -W \\ -Wy_c \\ -Wx_c \end{array}\right]}_{\overline{W}} \tag{8}$$

The top portion of this equation represents the n instances of the compatability equation (Equation 7). The bottom three rows represent the vertical, x-moment, and y-moment equations (Equations 3, 4, and 5). The vector on the right, $\overline{W}$, can be rewritten as follows

$$\overline{\mathbf{A}}\tilde{N} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ 0 & -W \\ -W & 0 \end{bmatrix}}_{\overline{\mathbf{W}}_x} \underbrace{\begin{bmatrix} x_c \\ y_c \end{bmatrix}}_{\overline{X}_c} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ -W \\ 0 \\ 0 \end{bmatrix}}_{\overline{W}_c} \tag{9}$$

$$\overline{\mathbf{A}}\tilde{N} = \overline{\mathbf{W}}_x \, \overline{X}_c + \overline{W}_c \tag{10}$$

Or, if we want to solve for $\tilde{N}$,

$$\tilde{N} = \overline{\mathbf{A}}^{-1}\overline{\mathbf{W}}_x \, \overline{X}_c + \overline{\mathbf{A}}^{-1}\overline{W}_c \tag{11}$$

Equation 11 gives us the normal forces $N_1 \cdots N_n$ as well as the plane parameters of the bottom of the parcel, a, b, and c. Since we are really just interested in the normal forces, we can multiply $\tilde{N}$ (which is a $n+3$ by 1 vector) by a n by $n+3$ matrix to "clip" off the plane parameters. This clip matrix appears as:

$$\overline{N} = \underbrace{\left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 0 & 0 \\ \vdots & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 \end{array}\right]}_{\overline{\mathbf{C}}} \tilde{N} \tag{12}$$

Then, Equation 11 becomes

$$\overline{N} = \overline{\mathbf{C}}\,\overline{\mathbf{A}}^{-1}\overline{\mathbf{W}}_x \, \overline{X}_c + \overline{\mathbf{C}}\,\overline{\mathbf{A}}^{-1}\overline{W}_c \tag{13}$$

4.2. Mask off Cells Under Parcel

In the previous section, we assumed that all the cells lay under the parcel. Since, in general, this is not true, we need a way of applying equation (11) to only those cells under the parcel. This can be accomplished through the use of a *mask* matrix. Each equation in the system of equations represented by equation (10) corresponds to an individual cell. When a particular cell is not under the parcel, its equation is meaningless. A mask matrix, $\overline{\mathbf{M}}$, can be contrived so as to remove the meaningless equations from the system of equations. The appropriate mask matrix can be constructed by removing columns from an $n+3$ by $n+3$ identity matrix. If the i^{th} cell is not under the parcel, the i^{th} column of the identity matrix is removed. Therefore, if m of the n total cells are under the parcel, $\overline{\mathbf{M}}$ will be of size $n+3$ by $m+3$.

As an example, suppose out of 4 cells, cell number 2 does not lie underneath the parcel. The mask matrix would be

$$\overline{\mathbf{M}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{14}$$

In this example, when $\tilde{N}$ is pre-multiplied by the transpose of $\overline{\mathbf{M}}$,

$$\overline{\mathbf{M}}^T \tilde{N} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} N_1 \\ N_2 \\ N_3 \\ N_4 \\ \hline c \\ b \\ a \end{array}\right] = \underbrace{\left[\begin{array}{c} N_2 \\ N_3 \\ N_4 \\ \hline c \\ b \\ a \end{array}\right]}_{\hat{N}} \tag{15}$$

which results in a vector, $\hat{N}$, containing only those normal forces under the parcel plus the three plane parameters. The full vector can be recovered (with zeros in the remaining places) by premultiplying by $\overline{\mathbf{M}}$. The mask matrix can also be used on the matrices and vectors in the equation $\overline{\mathbf{A}}$, $\overline{W}_c$, and $\overline{\mathbf{W}}_x$ to eliminate the appropriate terms and equations. Premultiplying $\overline{\mathbf{A}}$ by $\overline{\mathbf{M}}^{\mathrm{T}}$ removes the meaningless rows from the equation, and postmoltiplying by $\overline{\mathbf{M}}$ removes the terms corresponding to the outlying cells from the remaining equations. Premultiplying both $\overline{W}_c$ and $\overline{\mathbf{W}}_x$ by $\overline{\mathbf{M}}^{\mathrm{T}}$ eliminates the extra rows, leaving a set of equations, similar to, but smaller than, the original set. The equations appear as

$$\left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)\left(\overline{\mathbf{M}}^{\mathrm{T}} \tilde{N}\right) = \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{W}}_x\right) \overline{X}_c + \overline{\mathbf{M}}^{\mathrm{T}} \overline{W}_c \tag{16}$$

$\overline{\mathbf{M}}^{\mathrm{T}} \tilde{N} = \hat{N}$ is the vector containing only the forces supporting the parcel plus the three slopes. Solving for this yields

$$\hat{N} = \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{W}}_x\right) \overline{X}_c + \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1} \overline{\mathbf{M}}^{\mathrm{T}} \overline{W}_c \tag{17}$$

The vector of interest, however, is $\overline{N}$ which contains all the normal forces. Premultiplying $\hat{N}$ by $\overline{\mathbf{M}}$ will restore the normal forces which were previously removed, filling spaces with zeros. Also, the slopes of the parcel are not of interest, so premultiplying by $\overline{\mathbf{C}}$ will remove these terms. Therefore,

$$\overline{N} = \overline{\mathbf{C}}\,\overline{\mathbf{M}} \hat{N} \tag{18}$$

The complete set of supporting forces can now be solved for.

$$\overline{N} = \overline{\mathbf{C}}\,\overline{\mathbf{M}} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{W}}_x\right) \overline{X}_c + \overline{\mathbf{C}}\,\overline{\mathbf{M}} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1} \overline{\mathbf{M}}^{\mathrm{T}} \overline{W}_c \tag{19}$$

The vector $\overline{N}$ will contain the forces supporting the parcel and zeros in the places where the parcel does not rest.

4.3. Planar Dynamics Under Coulomb Friction

The full planar dynamics involve translation and rotation of the parcel. The horizontal forces are derived from the normal forces through the use of a coulomb friction law, where the horizontal force is proportional to the normal force in the direction of the wheel's spin. The horizontal forces are summed up to produce net forces and torques on the parcel which are simple functions of the parcel's position.

4.3.1. Translational Forces

As an example, we will examine a 2×2 array of cells with spacing d. The positions are defined relative to the center of the four cell giving the following position array

$$\overline{\mathbf{X}} = \begin{bmatrix} -\frac{d}{2} & \frac{d}{2} & -\frac{d}{2} & \frac{d}{2} \\ -\frac{d}{2} & -\frac{d}{2} & \frac{d}{2} & \frac{d}{2} \end{bmatrix} \tag{20}$$

Each column in this array gives the position of one cell. In this example, each cell pushes inward toward the center. Under coulomb friction, the force array (defining the direction each cell points) $\overline{\mathbf{F}}$ for this 2×2 array is

$$\overline{\mathbf{F}} = \begin{bmatrix} F_{x1} & F_{x2} & \cdots & F_{xn} \\ F_{y_1} & F_{y_2} & \cdots & F_{y_n} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \tag{21}$$

The sum of the horizontal forces from each cell can be found in the coulomb friction case by premultiplying the normal force vector, $\overline{N}$, by the coulomb force matrix, $\overline{\mathbf{F}}$, and the coefficient of friction, μ

$$\overline{f} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \mu \overline{\mathbf{F}}\, \overline{N} \tag{22}$$

$\overline{N}$ is found as shown in Section 4.2 and can be substituted in

$$\overline{f} = \underbrace{\mu \overline{\mathbf{F}}\, \overline{\mathbf{C}}\, \overline{\mathbf{M}} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\, \overline{\mathbf{M}}\right)^{-1} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{W}}_x\right)}_{\hat{\overline{\mathbf{a}}}} \overline{X}_c + \underbrace{\mu \overline{\mathbf{F}}\, \overline{\mathbf{C}}\, \overline{\mathbf{M}} \left(\overline{\mathbf{M}}^{\mathrm{T}} \overline{\mathbf{A}}\, \overline{\mathbf{M}}\right)^{-1} \overline{\mathbf{M}}^{\mathrm{T}} \overline{W}_c}_{\hat{\overline{\mathbf{b}}}} \tag{23}$$

where $\hat{\overline{\mathbf{a}}}$ is a constant (for a particular set of supports) 2×2 matrix, and $\hat{\overline{\mathbf{b}}}$ is a constant 2×1 vector. For simple, inward pointing fields, $\hat{\overline{\mathbf{a}}}$ has elements only on the diagonal, meaning that the y force on the parcel, f_y, only depends on y_c and the x force on the parcel, f_x, only depends on x_c This single-axis dependency of each force is only true for simple fields, $\overline{\mathbf{F}}$. More complicated fields may result in cross-dependency between the x and y directions.

4.3.2. Rotational Torque

In 2-D, the torque each cell applies to the parcel is the scalar cross product of the position vector between the parcel's center of mass, $\overline{X}_c$ and the point where the force is applied, $\overline{X}_i$, and the horizontal force vector from that point, $\overline{f}_i$. The i^{th} cell applies the following torque

$$\begin{aligned}
\tau_i &= \left(\overline{X}_i - \overline{X}_c\right) \times \overline{f}_i = \left(\begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} x_c \\ y_c \end{bmatrix} \right) \times \begin{bmatrix} f_{xi} \\ f_{y_i} \end{bmatrix} \\
&= \begin{bmatrix} x_i \\ y_i \end{bmatrix} \times \begin{bmatrix} f_{xi} \\ f_{y_i} \end{bmatrix} - \begin{bmatrix} x_c \\ y_c \end{bmatrix} \times \begin{bmatrix} f_{xi} \\ f_{y_i} \end{bmatrix} \\
&= \left(x_i f_{y_i} - y_i f_{xi}\right) - \left(x_c f_{y_i} - y_c f_{xi}\right)
\end{aligned} \tag{24}$$

In the case of coulomb friction, $f_{xi} = \mu F_{xi} N_i$ and $f_{y_i} = \mu F_{y_i} N_i$. Equation 24 becomes

$$\begin{aligned}
\tau_i &= \mu \left(x_i F_{y_i} N_i - y_i F_{xi} N_i\right) - \mu \left(x_c F_{y_i} N_i - y_c F_{xi} N_i\right) \\
&= \mu \left(x_i F_{y_i} - y_i F_{xi}\right) N_i - \mu \left(x_c F_{y_i} - y_c F_{xi}\right) N_i \\
&= \mu R_i N_i - \mu \left(\begin{bmatrix} x_c & y_c \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} F_{xi} \\ F_{y_i} \end{bmatrix} \right) N_i
\end{aligned} \tag{25}$$

where R_i is defined as $x_i F_{y_i} - y_i F_{xi}$. These terms form a $1 \times n$ vector, $\overline{R}$ which represents the amount of torque about the origin that each cell applies when multiplied by the normal force. The second term in this expressions corrects for the location of the center of mass of the parcel. $\overline{R}$ is a constant vector and can be pre-calculated from $\overline{\mathbf{X}}$ and $\overline{\mathbf{F}}$. For example, for the position and velocity direction matrices given in the 4-cell example in Equations 20 and 21, result in the following $\overline{R}$

$$\begin{aligned}
\overline{R} &= \begin{bmatrix} \left(-\frac{d}{2}\right) 1 - \left(-\frac{d}{2}\right) 1 & \frac{d}{2} 1 - \left(-\frac{d}{2}\right)(-1) & \left(-\frac{d}{2}\right)(-1) - \frac{d}{2} 1 & \frac{d}{2}(-1) - \frac{d}{2}(-1) \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{26}$$

The expression in Equation 25 must be summed up over all the cells in order to find the total torque. This can be done in vector form as follows

$$\tau = \mu \overline{R}\, \overline{N} - \mu\, \overline{X}_c^{\mathrm{T}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \overline{\mathbf{F}}\, \overline{N} \tag{27}$$

Now, the expression for $\overline{N}$ can be substituted in, giving

$$\begin{aligned}
\tau &= \mu\overline{R}\left(\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{W}}_x\right)\overline{X}_c + \overline{\mathbf{CM}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{AM}}\right)^{-1}\overline{\mathbf{M}}^{\mathrm{T}}\overline{W}_c\right) \\
&\quad -\mu\overline{X}_c^{\mathrm{T}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\overline{\mathbf{F}}\left(\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{W}}_x\right)\overline{X}_c + \overline{\mathbf{CM}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\overline{\mathbf{M}}^{\mathrm{T}}\overline{W}_c\right) \\
&= \mu\overline{R}\,\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{W}}_x\right)\overline{X}_c + \mu\overline{R}\,\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\overline{\mathbf{M}}^{\mathrm{T}}\overline{W}_c \\
&\quad -\mu\overline{X}_c^{\mathrm{T}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\overline{\mathbf{F}}\,\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{W}}_x\right)\overline{X}_c \\
&\quad -\mu\overline{X}_c^{\mathrm{T}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\overline{\mathbf{F}}\,\overline{\mathbf{C}}\,\overline{\mathbf{M}}\left(\overline{\mathbf{M}}^{\mathrm{T}}\overline{\mathbf{A}}\,\overline{\mathbf{M}}\right)^{-1}\overline{\mathbf{M}}^{\mathrm{T}}\overline{W}_c
\end{aligned} \tag{28}$$

This equation has four terms: a constant term, a term postmultiplied by $\overline{X}_c$, a term premultiplied by $\overline{X}_c^{\mathrm{T}}$, and a term pre- and postmultiplied by in $\overline{X}_c^{\mathrm{T}}$ and $\overline{X}_c$. While these equations are fairly complex, the form is reletively simple since, for a particular set of supports, most of the terms are constant, and reduce to 2×2 or smaller constant matrices. Equation 28 reduces to the form

$$\begin{aligned}
\tau &= \overline{X}_c^{\mathrm{T}}\hat{\overline{c}}\overline{X}_c + \overline{X}_c^{\mathrm{T}}\hat{\overline{d}} + \hat{\overline{e}}\overline{X}_c + \hat{f} \\
&= \overline{X}_c^{\mathrm{T}}\hat{\overline{c}}\overline{X}_c + \left(\hat{\overline{d}}^{\mathrm{T}} + \hat{\overline{e}}\right)\overline{X}_c + \hat{f}
\end{aligned} \tag{29}$$

where $\hat{\overline{c}}$ is a 2×2 matrix, $\hat{\overline{d}}$ is a 1×2 vector, $\hat{\overline{e}}$ is a 2×1 vector, and $\hat{f}$ is a scalar.

4.4. Planar Dynamics Under Viscous Friction

Under coulomb friction, the horizontal force ($\overline{f}_i$) from a cell was proportional to the normal force, but only dependent on the direction of the wheel's velocity. With a simple viscous (velocity dependent) friction law, the horizontal force is proportional to both the normal force and the difference between the velocity of the wheel and the velocity of the parcel at the point of the cell. This velocity difference is a function of both the translational velocity of the parcel ($\dot{\overline{X}}_c$), the velocity of the wheel ($\overline{V}_i$), the rotation speed of the parcel about its center of mass (ω), and the position difference between the cell and the center of mass ($\overline{X}_i - \overline{X}_c$).

$$\overline{f}_i = \begin{bmatrix} f_{xi} \\ f_{yi} \end{bmatrix} = \mu\left(\overline{V}_i - \dot{\overline{X}}_c + \omega\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\left(\overline{X}_i - \overline{X}_c\right)\right)N_i \tag{30}$$

4.4.1. Translational Forces

The horizontal force from each cell is summed up over all the cells. This can be done by forming a wheel velocity matrix, $\overline{\mathbf{V}}$ similar to the coulomb matrix $\overline{\mathbf{F}}$ which contains the x and y velocities of all the wheels in the array. This matrix can then be used to sum up the horizontal forces:

$$\overline{f} = \sum_{i=1}^{n}\overline{f}_i = \mu\left(\overline{\mathbf{V}} - \dot{\overline{X}}_c\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} + \omega\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\left(\overline{\mathbf{X}} - \overline{X}_c\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}\right)\right)\overline{N} \tag{31}$$

$$= \overline{\mathbf{V}}\,\overline{N} - \dot{\overline{X}}_c\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}\overline{N} + \omega\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\left(\overline{\mathbf{X}}\,\overline{N} - \overline{X}_c\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}\overline{N}\right) \tag{32}$$

Note that $\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}\overline{N}$ is the sum of the normal forces, which is the parcel weight, W. Therefore, the last two terms in the parentheses represent Equation 1, and are identically zero. Hence, the net horizontal force is not a function of the parcel's rotation speed and Equation 32 becomes

$$\overline{f} = \mu\overline{\mathbf{V}}\,\overline{N} - \mu\dot{\overline{X}}_c W \tag{33}$$

which is almost identical to the coulomb case plus a linear damping term proportional to weight. This term is always dissipative since weight is always positive. The difference from the coulomb case is that the terms in the matrix $\overline{V}$ can have arbitrary magnitude, while the coulomb matrix $\overline{\mathbf{F}}$ has terms either equal to zero or of unit magnitude.

4.4.2. Rotational Torque

To calculate the torque in the viscous case, we can start with Equation 24, but now we substitute the new expression for $\overline{f}_i$.

$$\begin{aligned}
\tau_i &= \mu\left(x_i\left(V_{y\,i}-\dot{y}_c-\omega\left(x_i-x_c\right)\right)-y_i\left(V_{x\,i}-\dot{x}_c+\omega\left(y_i-y_c\right)\right)\right)N_i - \\
&\quad \mu\left(x_c\left(V_{y\,i}-\dot{y}_c-\omega\left(x_i-x_c\right)\right)-y_c\left(V_{x\,i}-\dot{x}_c+\omega\left(y_i-y_c\right)\right)\right)N_i & (34)\\
&= \mu {R^V}_i N_i - \mu\begin{bmatrix} x_c & y_c \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} V_{x\,i} \\ V_{y\,i} \end{bmatrix} N_i \\
&\quad -\mu\left(x_i\dot{y}_c - y_i\dot{x}_c\right)N_i + \mu\left(x_c\dot{y}_c - y_c\dot{x}_c\right)N_i \\
&\quad -\mu\omega\left(\left(x_i-x_c\right)\left(x_i-x_c\right)N_i + \left(y_i-y_c\right)\left(y_i-y_c\right)N_i\right) & (35)\\
&= \mu {R^V}_i N_i - \mu\begin{bmatrix} x_c & y_c \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} V_{x\,i} \\ V_{y\,i} \end{bmatrix} N_i \\
&\quad -\mu\left(\begin{bmatrix} x_i & y_i \end{bmatrix} - \begin{bmatrix} x_c & y_c \end{bmatrix}\right)N_i\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\dot{\overline{X}}_c \\
&\quad -\mu\omega\left(x_i\left(x_i-x_c\right)N_i + y_i\left(y_i-y_c\right)N_i - y_c\left(y_i-y_c\right)N_i - x_c\left(x_i-x_c\right)N_i\right) & (36)
\end{aligned}$$

where ${R^V}_i$ is defined as $x_i V_{y\,i} - y_i V_{x\,i}$ (which defines $\overline{R}^V$). The torque from each cell in Equation 36 must be summed up over all the cells. The summation of the terms containing $\left(x_i-x_c\right)N_i$ and $\left(y_i-y_c\right)N_i$ (i.e. the middle line and the last two terms in the last line of Equation 36) will result in terms similar those terms in parentheses in Equation 32 and sum up to zero. This leaves us with the following expression for τ

$$\tau = \mu\overline{R}^V\,\overline{N} - \mu{\overline{X}_c}^T\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{V}\,\overline{N} - \mu\omega\left(\underbrace{\begin{bmatrix} {x_1}^2+{y_1}^2 & {x_2}^2+{y_2}^2 & \cdots & {x_n}^2+{y_n}^2 \end{bmatrix}}_{\overline{X}^2}\overline{N} - {\overline{X}_c}^T\mathbf{X}\,\overline{N}\right) \tag{37}$$

The matrix $\overline{X}^2$ represents the second moment of position of each of the cells. The last term contains $\mathbf{X}\,\overline{N}$ which, from Equations 4 and 5, is equal to $\overline{X}_c W$. The torque can then be represented as

$$\tau = \mu\overline{R}^V\,\overline{N} - \mu{\overline{X}_c}^T\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{V}\,\overline{N} - \mu\omega\left(\overline{X}^2\overline{N} - W{\overline{X}_c}^T\overline{X}_c\right) \tag{38}$$

This expression is very similar to the coulomb case, with an extra rotational damping term related to the second moment of area of the parcel. This damping term is a function of parcel position because the locations of the damping forces affects their ability to produce a torque. The term multiplying $-\mu\omega$ was formed from the sum over all the cells of the last line in Equation 35, which consists entirely of positive terms. Therefore, the damping always opposes the rotational motion of the parcel.

4.5. Motion Prediction

The equations derived for horizontal forces and torque on the parcel reduce the multiple support, multiple friction contact model to a small set of constants which apply for all parcel positions over a particular set of supports. In order to use this to predict the motion of a prcel for a given cell velocity field, the following procedure is required:

- Compute which cells lie under the parcel.
- Form the appropriate mask matrix.
- Compute the constants used to calulate forces.
- Integrate the motion, checking the location of supports.
- When the supports change, recompute the constants and continue.

Because the position dependency of the forces and torques is reduced to a small set of constants, integrating the motion is very simple, allowing fast piecewise simulation of the motion. The more difficult calculations are only necessary when the supports change.

5. CONCLUSIONS AND FUTURE WORK

The concept of the Virtual Vehicle as a transport and manipulation system using distributed actuation was described. This system has many advantages over traditional systems. The main body of this paper involves the computation of net forces and torque on a parcel riding on the Virtual Vehicle when the wheels of each cell spin at constant speeds.

The equations derived in Section 4 provide a concise method of calculating the forces and torques applied to a parcel riding on a set of many sliding friction contacts. Two friction models were considered, coulomb and viscous. The coulomb case returned very simple dynamics, where the load distribution and contact forces reduce to a small set of constants. The viscous case produced similar results, where the viscous friction model returned additional dissipative terms.

These calculations apply only while the parcel rests on a single subset of cells. In order to predict the motion as a parcel changes supports over many cells, it is necessary to reapply the mask matrix as each set of supports is reached. It is a simple matter, however, to simulate the parcel's motion between transitions.

In the future, this piecewise local dynamic representation will be generalized, possibly in an approximate form, to determine global dynamics. Potential function approximations of the true force field may be used. Also, the induced force field must be examined for stability and equilibria of the parcel. The design problem must be also solved, and that is how to design wheel velocity fields to induce desired motions on the parcel to control its position and orientation. FInally, in order to better model the contact between the pacel and he wheels, a stick-slip friction model will be used, requiring another maks matrix based on normal force and static friction thresholds.

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