Velocity Field Design for the Modular Distributed Manipulator System (MDMS)

Jonathan Luntz, *Carnegie Mellon University, Pittsburgh, PA, USA*
William Messner, *Carnegie Mellon University, Pittsburgh, PA, USA*
Howie Choset, *Carnegie Mellon University, Pittsburgh, PA, USA*

The Modular Distributed Manipulator System *(MDMS) is a novel materials handling system which is a fixed array of actuated wheels capable of inducing arbitrary motions in the plane. This work develops control laws for transporting and manipulating macroscopic-scale objects (parcels) that rest on the array. The dynamics of parcel transport and manipulation lead to the necessary understanding of such laws. The dynamics are based on an exact discrete representation of the system, unlike other methods where a continuity assumption is made. Given a particular set of wheel speeds, the dynamics of a parcel's motion are derived by examining the distribution of weight among the supporting cells subject to a friction model governing the parcel-wheel contact. The resulting relationship between parcel behavior and wheel speeds can be inverted, therefore wheel speeds can be calculated from desired parcel behavior.*

1 Introduction

The *Modular Distributed Manipulator System* (MDMS) transfers, as well as manipulates, objects in the plane, enhancing applications such as flexible manufacturing and package handling systems. Flexible manufacturing requires that working part paths be arbitrarily specified and that these paths be readily updated by a "factory programmer." Package handling systems, such as airport baggage handling, require parcels to be sorted, oriented, and transported to one of several locations.

Two existing methods of material transfer are conveyor belts and robotic manipulators. The conveyor belts transfer many large heavy objects for long distances, but lack the ability to precisely orient objects.

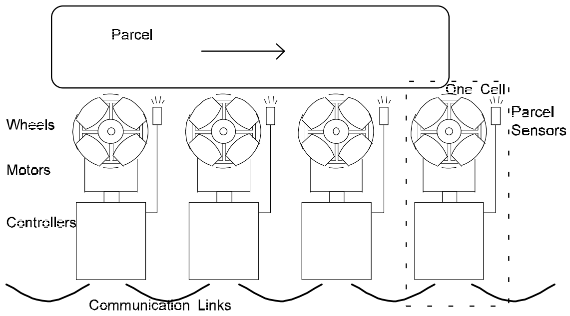

Figure 1: *A few manipulator cells carrying a parcel*

Furthermore, conveyor belts cannot re-order objects, nor are they well suited to merging objects from different sources. Robots, on the other hand, are able to precisely position and orient objects, but only one at a time, and are limited by strength and reach. The MDMS combines the benefits of conveyor and robotics transport systems while overcoming their limitations.

This alternative method comprises a fixed array of actuators that cooperate to handle objects. In the prototype system, each cell consists of a pair of actuators whose combined action can effect force in any planar direction to a parcel resting on top of the array. Furthermore, the cells contain binary sensors that detect the presence of an object. See Figure 1.

In this system, the parcels are significantly larger than each cell; several cells support a single parcel that can be made to translate and rotate in the plane. Since sensing and actuation are distributed, each of many parcels can be manipulated independently, appearing as if each parcel were carried by a separate vehicle (Figure 2).

Implementation of this system requires the develop-

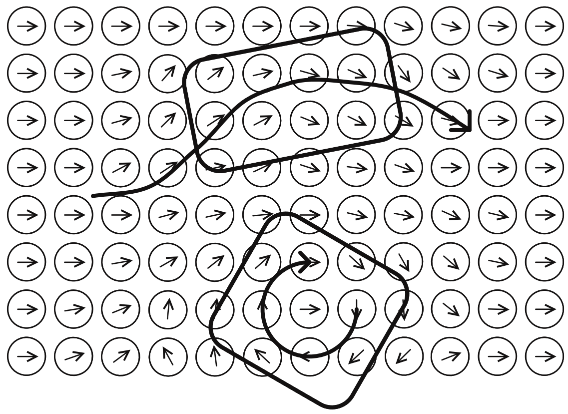

Figure 2: *The MDMS: Several parcels can be translated and rotated, independently.*

ment of control laws and policies to provide the desired motions of parcels. These control laws must reflect a physical system, such as the one described in Section 2. These laws build upon an already vast literature of manipulation and actuator arrays, which are described in Section 3.

The control laws are based on the dynamics of a parcel being transported by the MDMS. These dynamics are derived in Section 4 by first calculating the forces which support a parcel resting on an arbitrary set of cells. A friction law is then applied to compute the net forces and moments acting on the parcel.

Section 4 describes the dynamics, given a specific set of wheel velocity. Section 5 describes the inverse problem: given desired parcel dynamics, compute the necessary wheel velocities to attain these dynamics uniformly over the entire array. The resulting field is analyzed for its translational and rotational properties. Under certain assumptions under parcel shape and array symmetry, an object can be positioned and oriented within the resolution of the array to symmetry.

2 Application and Prototype Configuration

For many applications, a dedicated robot or conveyor is the simplest and most appropriate solution. There are cases, however, where additional flexibility, redundancy, modularity and reconfigurability are required, giving the MDMS significant advantages over existing systems.

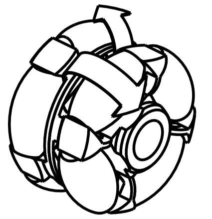

Figure 3: *Roller wheel.*

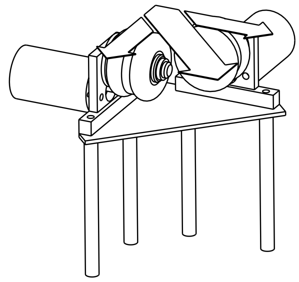

Figure 4: *Prototype cell.*

The MDMS can be used in conjunction with traditional robots and conveyor belts to form hybrid systems. For example, in airport baggage handling, long conveyors can be used to transport parcels over long distances while MDMS arrays can be installed at conveyor junctions to sort and to re-direct parcel traffic. In flexible manufacturing, the MDMS can be used to transport objects between robot workspaces where simple robots are used for object fixturing.

Mechanical Configuration. A prototype system was built consisting of a small array of cells capable of transporting objects about the size of a bread box. Each cell consists of a pair of orthogonally oriented motorized roller wheels (Figures 3 and 4) which are capable of producing a force perpendicular to their axes, while allowing free motion parallel to their axes. Each wheel is driven through a gear reduction by a small DC motor. Each of these cells is attached to a breadboard base (Figure 5) to form a regular array. Currently, the prototype system consists of 18 cells which can be arranged either in a 1-D or a 2-D grid, shown in the photograph in Figure 6.

Electronic Configuration. Each cell is controlled by a MC68HC11-based single-board computer. Distributed control such as this is a relatively underdeveloped and interesting control paradigm and is well suited for the MDMS because it would become impractical or impossible for a single computer to control

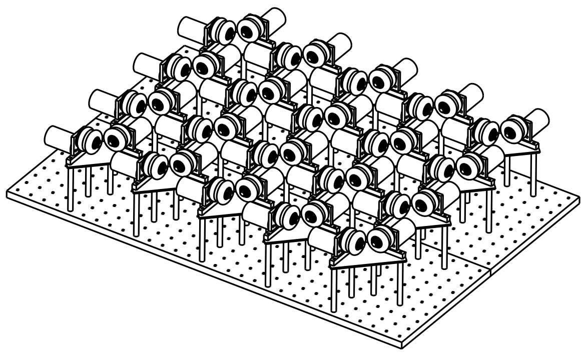

Figure 5: *Two-dimensional array of cells.*

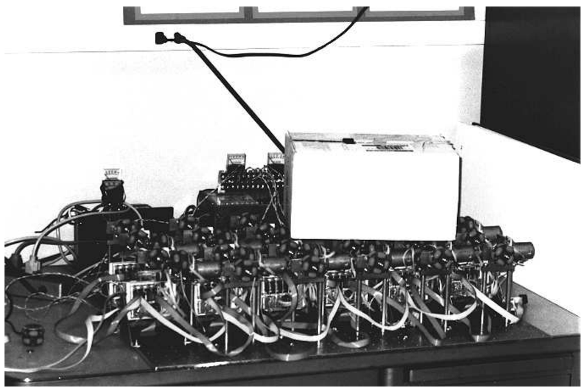

Figure 6: *Current experimental setup.*

hundreds or possibly thousands of cells. The speed of the wheels is measured by a quadrature encoder. The presence of a parcel is sensed by a phototransistor which detects the shadow from the overhead lights. The motor is driven by pulse width modulation using an H-bridge circuit.

Network Configuration. An important aspect of our control scheme is the communication between cells. Each controller has an RS232 communications port which is multiplexed between each of the four neighboring cells, requiring active selection of neighbors. Handshaking lines are used to alert a neighboring cell of a pending message. The cells communicate with their neighbors in a consistent pattern, automatically synchronizing communication.

This communication scheme is more appropriate for our system than a more standard shared-bus local area network. The multiplexed network supports many simultaneous communications, whereas the shared bus only allows one at a time. Our network requires only relies on local information without addressing whereas a standard network requires each cell to be aware of its address and is limited by a maximum address space. Messages can be passed along the array if necessary. Messages are generated using a communication language which involves the construction of short "sentences" made up of labeled pieces of data, or words, punctuated by a checksum.

3 Prior Work

Mason and Erdmann [1, 3] provided a radical alternative to standard robotic manipulation by significantly simplifying the robot manipulator and developing manipulation algorithms for these low-degree-of-freedom sensorless mechanisms. Goldberg [4] developed an algorithm which orients to symmetry a part with a sequence of gripper open, close, and rotation operations without sensor information. This sequence of operations is termed a *squeeze*.

Böhringer, Donald, et. al. [2] applied this type of sensorless manipulation to an array of micromechanical actuators which were used to manipulate very small objects. Whereas Goldberg used a single centralized manipulator to orient objects, Böhringer used distributed actuation, but in both methods the control is centralized. The Böhringer system differs from the MDMS in that (i) it is on a small scale, (ii) the control is centralized, and (iii) the manipulation is entirely sensorless. It is worth noting that although their work has motivated some of the authors' research, their work uses quasi-static analysis, and ignores dynamics because of the small scale application.

Recently, Kavraki [5] supplied further analysis of microactuated systems using elliptical potential fields to orient an object to symmetry without sensors. Potential functions are used to predict stable configurations of objects. Continuous potential functions adequately approximate Kavraki's and Böhringer's systems, because many micro-sized cells provide essentially continuous fields. The MDMS, however, is highly discrete such potential fields can not be used for motion planning. Only a small number of cells support an object.

In a previous paper by the authors [6] the first step was taken in examining the dynamics of a parcel carried by the MDMS, where the one dimensional motion of the parcel along the array of cells was considered. In that paper, the forces between each cell and the parcel were calculated, and both a coulomb and a viscous-like friction law were considered. The resulting parcel dynamics are that of a simple or damped harmonic oscillator, where the frequency, center of oscillation, and damping constant are parameters which change as the parcel shifts from one set of supports to another. This simple oscillator behavior was also observed in the prototype system. This work was extended into two dimensions in a more recent paper by the authors [7]. Translational forces and rotational torques are calculated as a function of parcel position. Similar mass-spring-damper behavior in the plane was observed.

This paper upgrades the authors' previous work in two dimensions by significantly reducing the amount of calculation necessary to calculate the forces and torques, allowing for the inversion of the dynamics. A wheel velocity field is thus generated which produce desired parcel dynamics with a single equilibrium position regardless of symmetry and cell resolution, and, under certain assumptions, a single orientational position (within symmetry) to the resolution of the cells.

The ultimate goal of this work is to be able to predict and control the overall motion of one or more parcels with two-dimensional translation with rotation in the plane. The work on the MDMS continues a trend in the previous work, where a transition was made from standard, centralized manipulation with sensors to a minimalist, sensorless (but still centralized) approach to a distributed sensorless approach on a microscopic scale. The MDMS again focuses on a macroscopic scale and also includes sensors and feedback. This trend is shown in Figure 7.

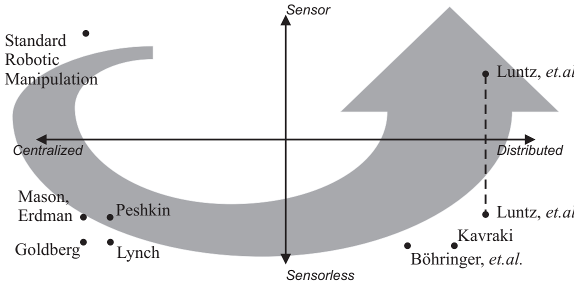

Figure 7: *Trends in Manipulation*

4 Dynamics of Manipulation

Towards the goal of representing the dynamics of handling a parcel in the plane with an array of actuators, we will consider the dynamics of an array of cells transporting and rotating a parcel in the plane while it rests on a single arbitrary set of cells. The following assumptions are made:

- Each orthogonally oriented pair of wheels acts as a single support.
- Supports act as springs to support the parcel.
- The bottom of the parcel is flat.
- The speed of each wheel is constant.
- Horizontal force between each wheel and the parcel is due to sliding friction.
- Viscous friction (proportional to speed) exists between the wheels and parcel which is also proportional to the normal force.

In order to compute the horizontal translation and rotation dynamics of the parcel, first the equilibrium of the parcel and constitutive relations for the supports are used to compute the normal forces supporting the parcel. Then, a friction law between the parcel and wheels is applied to determine the net horizontal forces and moments The result will be a net force and torque acting on the parcel as a function of the parcel's position. The problem will be considerably simplified by the structure of the mathematics and by several convenient mathematical identities.

Notation: Because of the large number of variables used, this paper will employ a consistent notation. Scalar variables will appear in normal math font (e.g.

s), vectors will appear in normal math font with an arrow (e.g. $\vec{v}$), and matrices will appear in bold font (e.g. $\mathbf{m}$). Subscripts x and y will indicate x and y components, and subscript i will indicate the i^{th} cell. For example, $\mathbf{V}$ is a matrix made up of velocity (column) vectors $\vec{V}_i$ for each cell, with components V_{x_i} and V_{y_i}. $\mathbf{V}$ can also be said to be made up of component (row) vectors $\vec{V}_x$ and $\vec{V}_y$ listing the velocity components for all the cells.

4.1 Normal Forces

In order to solve for the n forces supporting the parcel, the equilibrium of the parcel in both the vertical (z) direction and in rotation about the x and y axes must be considered. Since equilibrium only provides three equations, and we have n supports, the system is statically indeterminant, and flexibility in the supports (or equivalently, in the parcel surface, as in the prototype) needs to be considered to solve for the remaining $n-3$ forces. These $n-3$ equations will be referred to as *compatibility* equations. The n equations will be represented in matrix form, and can be used to solve for the vector of n normal forces, $\vec{N}$.

4.1.1 Parcel equilibrium

Consider n cells arranged arbitrarily in the x-y plane having coordinates as entries of the matrix below.

$$\mathbf{X} = \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \begin{bmatrix} x_1 \dots x_n \\ y_1 \dots y_n \end{bmatrix} = \begin{bmatrix} \vec{X}_1 \dots \vec{X}_n \end{bmatrix} \tag{1}$$

A parcel, whose center of mass is located at $\vec{X}_{cm} = \begin{bmatrix} x_{cm} & y_{cm} \end{bmatrix}^{\mathrm{T}}$ resting on n of these cells, is supported by vertical normal forces

$$\vec{N} = \begin{bmatrix} N_1 & N_2 & N_3 & \dots & N_n \end{bmatrix}. \tag{2}$$

The parcel must be in equilibrium in the vertical direction and in rotation about the x and y axes. The vertical equilibrium of the parcel, with weight W, requires that

$$\sum_{i=1}^{n} N_i = W = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \end{bmatrix} \vec{N}^{\mathrm{T}}. \tag{3}$$

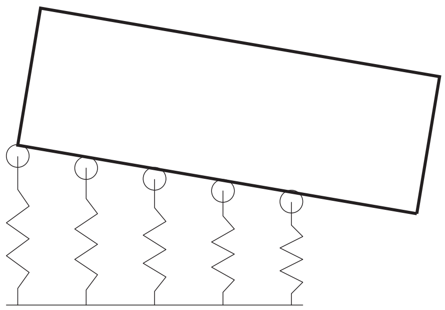

Figure 8: *Flexible supports in many-cell case.*

Rotational equilibrium about the x and y axes require that the moments induced by the normal forces about any point (in this case, the arbitrarily located origin of our coordinate system) sum to the moment about this point induced by the weight of the parcel. The moment equilibrium about the x and y axes, respectively, requires that

$$\sum_{i=1}^{n} N_i y_i = \vec{y}\,\vec{N}^{\mathrm{T}} = W y_{cm}, \text{ and} \tag{4}$$

$$\sum_{i=1}^{n} N_i x_i = \vec{x}\,\vec{N}^{\mathrm{T}} = W x_{cm}. \tag{5}$$

At this point in the development, there are n unknowns (the elements of $\vec{N}$), but only three equations (3,4, and 5) from equilibrium.

4.1.2 Compatibility

To solve for the remaining $n-3$ forces, flexibility in the system must be considered. A simple model of this flexibility is shown in side view in Figure 8. Each support is assumed to be a spring, with Hooke's Law ($N_i = K_s \Delta z_i$) representing the compression of the i^{th} cells under a normal load. Physically, this flexibility can be thought of as a flexible suspension under each wheel, as will be included in the next generation of our hardware array. In the current system, an equivalent flexibility is apparent in the bottom of the parcel, either in the cardboard of a box, or a foam pad on which the box rests.

Assuming the bottom of the box is nominally flat, the flexible cells conform to the bottom of the box to distribute the weight. All the cells (under the parcel) lie in this plane. The plane of the bottom of the parcel can be represented as

$$z = a'y + b'x + c'. \tag{6}$$

Each cell will obey the following displacement relation.

$$\begin{aligned} \Delta z_i = N_i/K_s &= a'y_i + b'x_i + c' \\ \Rightarrow N_i &+ ay_i + bx_i + c = 0 \end{aligned} \tag{7}$$

4.1.3 Combining Equations

Equations 3, 4, and 5 along with n instances of Equation 7 supply $n+3$ equations and $n+3$ unknowns (n N_i's and 3 plane parameters, a, b, and c). This system of equations can be written in matrix form.

$$\underbrace{\left[\begin{array}{ccc|ccc} 1 & \dots & 0 & 1 & x_1 & y_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & 1 & x_n & y_n \\ \hline 1 & \dots & 1 & 0 & 0 & 0 \\ x_1 & \dots & x_n & 0 & 0 & 0 \\ y_1 & \dots & y_n & 0 & 0 & 0 \end{array}\right]}_{\mathbf{A}} \underbrace{\left[\begin{array}{c} N_1 \\ \vdots \\ N_n \\ \hline c \\ b \\ a \end{array}\right]}_{\vec{N}^{\mathrm{T}}_{abc}} = \underbrace{\left[\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline W \\ Wx_c \\ Wy_c \end{array}\right]}_{\vec{W}} \tag{8}$$

The top portion of this equation represents the n instances of the compatibility equation (Equation 7). The bottom three rows represent the vertical, x-moment, and y-moment equations (Equations 3, 4, and 5). This equation can be inverted to solve for $\vec{N}_{abc}$ (which contains $\vec{N}$ and a, b, and c.)

$$\vec{N}^{\mathrm{T}}_{abc} = \mathbf{A}^{-1}\vec{W} \tag{9}$$

4.1.4 Solving for Normal Forces

In order to solve for the normal forces in $\vec{N}$, $\mathbf{A}$ must be inverted. Since $\mathbf{A}$ is symmetric and very structured, this can readily be done. Define a matrix $\mathbf{B}$.

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ y_1 & y_2 & y_3 & \dots & y_n \end{bmatrix}. \tag{10}$$

The matrix $\mathbf{A}$ can be written in block matrix form.

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{I}_{n\times n} & \mathbf{B}^{\mathrm{T}} \\ \hline \mathbf{B} & \mathbf{0}_{3\times 3} \end{array}\right] \tag{11}$$

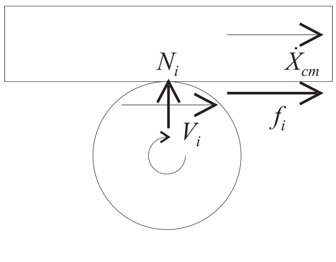

Figure 9: *Interaction between wheel and parcel.*

The inverse of the matrix $\mathbf{A}$ exists if $\mathbf{B}$ has rank 3 (which is true for general cell positions). The expression for the inverse is

$$\mathbf{A}^{-1} = \left[\begin{array}{c|c} \mathbf{I}_{n\times n} - \mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{B} & \mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \\ \hline \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{B} & -\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \end{array}\right]. \tag{12}$$

The derivation of the inverse is omitted, but the result can easily be verified by multiplying $\mathbf{A}$ by $\mathbf{A}^{-1}$.

Since $\vec{W}$ only multiplies nonzero elements into the right side of $\mathbf{A}^{-1}$, and only the slopes (a, b, and c) result from the lower portion of $\mathbf{A}^{-1}$, only the upper right partition of $\mathbf{A}^{-1}$ is used to calculate $\vec{N}$.

$$\begin{aligned} \vec{N}^{\mathrm{T}} &= \mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \begin{bmatrix} W \\ Wy_{cm} \\ Wy_{cm} \end{bmatrix} \\ &= W\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{X}_{cm} \right). \end{aligned} \tag{13}$$

4.2 Planar Dynamics Under Viscous Friction

The full planar dynamics involve translation and rotation of the parcel. The horizontal forces are derived from the normal forces through the use of a viscous-type friction law. The horizontal force from each cell $\vec{f}_i$ is proportional to a coefficient of friction μ, that cell's normal force N_i, and the vector difference between the velocity of the wheel and the velocity of the parcel at the point of the cell. This velocity difference is a function of both the translational velocity of the parcel $\dot{\vec{X}}_{cm}$, the velocity of the wheel $\vec{V}_i$, the rotation speed of the parcel about its center of mass ω, and the position difference between the cell and the center of mass $\vec{X}_i - \vec{X}_{cm}$.

$$\vec{f}_i = \mu\left(\vec{V}_i - \dot{\vec{X}}_{cm} + \omega\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left(\vec{X}_i - \vec{X}_{cm}\right)\right)N_i \tag{14}$$

4.2.1 Translational Forces

The horizontal force from each cell is summed up over all the cells. This can be done by defining a wheel velocity matrix $\mathbf{V}$ as

$$\mathbf{V} = \begin{bmatrix} V_{1_x} & V_{2_x} & \dots & V_{n_x} \\ V_{1_y} & V_{2_y} & \dots & V_{n_y}\end{bmatrix} \tag{15}$$

$\mathbf{V}$, along with the cell position matrix $\mathbf{X}$, can be used to sum up the horizontal forces.

$$\begin{aligned}\vec{f} &= \sum_{i=1}^{n}\vec{f}_i = \mu\left(\mathbf{V} - \dot{\vec{X}}_{cm}[1\,1\dots1]\right.\\ &\quad\left.+\omega\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left(\mathbf{X} - \vec{X}_{cm}[1\,1\dots1]\right)\right)\vec{N}^{\mathrm{T}}\\ &= \mu\mathbf{V}\ \vec{N}^{\mathrm{T}} - \mu\dot{\vec{X}}_{cm}[1\,1\dots1]\vec{N}^{\mathrm{T}}\\ &\quad+\omega\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left(\mathbf{X}\ \vec{N}^{\mathrm{T}} - [1\,1\dots1]\vec{N}^{\mathrm{T}}\right)\end{aligned} \tag{16}$$

Note that $[1\,1\dots1]\vec{N}^{\mathrm{T}}$ is the sum of the normal forces, which is the parcel weight, W. Therefore, the last two terms in the parentheses are identically zero from Equations 4 and 5. Hence, the net horizontal force is not a function of the parcel's rotation speed and Equation 16 becomes

$$\vec{f} = \mu\mathbf{V}\ \vec{N}^{\mathrm{T}} - \mu\dot{\vec{X}}_{cm}W, \tag{17}$$

The second term in this equation is a linear damping term which is always dissipative since weight is always positive. $\vec{N}$ in the first term can be substituted from Equation 13.

$$\begin{aligned}\vec{f} &= \underbrace{\mu W\mathbf{V}\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{k_s}}\vec{X}_{cm}\\ &\quad\underbrace{+\mu W\mathbf{V}\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}}_{\vec{f}_o} - \mu\dot{\vec{X}}_{cm}W,\end{aligned} \tag{18}$$

where $\mathbf{k_s}$ is a constant 2×2 matrix and $\vec{f}_o$ is a constant 2×1 vector. The matrix $\mathbf{k_s}$ is essentially a matrix of spring constants, since it specifies force as a linear function of position. The vector $\vec{f}_o$ is an offset force.

4.2.2 Rotational Torque

In two dimensions, the torque each cell applies to the parcel is the scalar cross product of the position vector of the point of application of the force, $\vec{X}_i$, relative to the parcel center of mass, $\vec{X}_{cm}$, and the horizontal force vector from that point, $\vec{f}_i$. With viscous friction, substitute in Equation 14 for $\vec{f}_i$. Applying the matrix definition of scalar cross products, the i^{th} cell applies the following torque.

$$\begin{aligned}\tau_i &= \mu\left(\vec{X}_i - \vec{X}_{cm}\right)\times\left(\vec{V}_i - \dot{\vec{X}}_{cm} + \omega\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left(\vec{X}_i - \vec{X}_{cm}\right)\right)N_i\\ &= \mu\vec{X}_i\times\vec{V}_iN_i - \mu\vec{X}_{cm}\times\vec{V}_iN_i - \mu\left(\vec{X}_i - \vec{X}_{cm}\right)\times\dot{\vec{X}}_{cm}N_i\\ &\quad+\omega\mu\left(\vec{X}_i - \vec{X}_{cm}\right)^{\mathrm{T}}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left(\vec{X}_i - \vec{X}_{cm}\right)N_i\end{aligned} \tag{19}$$

This expression can be reduced by defining $R_i = \vec{X}_i\times\vec{V}_i$ (which defines $\vec{R}$) and $\mathcal{X}_i = \vec{X}_i^{\mathrm{T}}\vec{X}_i$ (which defines $\vec{\mathcal{X}}$). Also, multiplying the cross product matrices yields $-\mathbf{I}$. In addition, use the fact that $\vec{X}_i^{\mathrm{T}}\vec{X}_{cm} = \vec{X}_{cm}^{\mathrm{T}}\vec{X}_i$, and that reversing the order of a cross product changes sign.

$$\begin{aligned}\tau_i &= \mu R_iN_i - \mu\vec{X}_{cm}\times\vec{V}_iN_i + \mu\dot{\vec{X}}_{cm}\times\left(\vec{X}_i - \vec{X}_{cm}\right)N_i\\ &\quad-\omega\mu\left(\vec{X}_i - \vec{X}_{cm}\right)^{\mathrm{T}}\left(\vec{X}_i - \vec{X}_{cm}\right)N_i\\ &= \mu R_iN_i - \mu\vec{X}_{cm}\times\vec{V}_iN_i + \mu\dot{\vec{X}}_{cm}\times\left(\vec{X}_i - \vec{X}_{cm}\right)N_i\\ &\quad-\omega\mu\left(\mathcal{X}_iN_i - \vec{X}_{cm}^{\mathrm{T}}\vec{X}_iN_i - \vec{X}_{cm}^{\mathrm{T}}\left(\vec{X}_i - \vec{X}_{cm}\right)N_i\right)\end{aligned} \tag{20}$$

Note that the term multiplying ω is positive since N_i is always positive as is $\left(\vec{X}_i - \vec{X}_{cm}\right)^{\mathrm{T}}\left(\vec{X}_i - \vec{X}_{cm}\right)$.

The torque from each cell in Equation 20 must be summed up over all the cells. This can be done vectorially by taking advantage of the inner product.

$$\tau = \sum \tau_i = \mu \vec{R}\vec{N}^{\mathrm{T}} - \mu \vec{X}_{cm} \times \mathbf{V}\vec{N}^{\mathrm{T}} + \mu \dot{\vec{X}}_{cm} \times \left(\mathbf{X}\vec{N}^{\mathrm{T}} - \vec{X}_{cm} [1\,1 \ldots 1] \vec{N}^{\mathrm{T}} \right) - \omega\mu \left(\vec{\mathcal{X}}\vec{N}^{\mathrm{T}} - \vec{X}_{cm}^{\mathrm{T}} \mathbf{X}\vec{N}^{\mathrm{T}} - \vec{X}_{cm}^{\mathrm{T}} \left(\mathbf{X}\vec{N}^{\mathrm{T}} - \vec{X}_{cm} [1\,1 \ldots 1] \vec{N}^{\mathrm{T}} \right) \right) \quad (21)$$

Some terms can be eliminated by examining this equation. $\mathbf{X}\vec{N} = W\vec{X}_{cm}$, and $[1\,1 \ldots 1]\vec{N} = W$ from parcel equilibrium (Equations 3, 4, and 5). Therefore, $\mathbf{X}\vec{N} - \vec{X}_{cm}[1\,1 \ldots 1]\vec{N} = 0$ and the torque can be expressed as

$$\tau = \mu \vec{R}\vec{N} - \mu \vec{X}_{cm} \times \mathbf{V}\vec{N} - \omega\mu \left(\vec{\mathcal{X}}\vec{N} - W \vec{X}_{cm}^{\mathrm{T}} \vec{X}_{cm} \right) \quad (22)$$

The last term in this expression is $-\omega$ multiplied by a positive term (from Equation 20) and hence always opposes the parcel's rotation and is dissipative. The expression for the normal forces can be substituted in to give an expression for the moments acting on the parcel as a function of position and rotational speed.

$$\tau = \underbrace{\mu W \vec{R}\mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}} \right)^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\tau_o} + \underbrace{\mu W \vec{R}\mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}} \right)^{-1} \begin{bmatrix} 0\,0 \\ 1\,0 \\ 0\,1 \end{bmatrix}}_{\vec{k}_{s_\tau}} \vec{X}_{cm} + \vec{X}_{cm} \times \left(\vec{f}_o + \mathbf{k_s} \vec{X}_{cm} \right) - \mu\omega \left(\vec{\mathcal{X}}\vec{N} - W \vec{X}_{cm}^{T} \vec{X}_{cm} \right) \quad (23)$$

where $\mathbf{k_s}$ and $\vec{f}_o$ are the spring and offset constants from the translational dynamics, $\vec{k}_{s_\tau}$ is a 1×2 constant vector relating torque to position, and τ_o is a scalar constant torque. Note that nothing in the previous mathematics involved the orientation of the parcel, and hence, while the parcel rests on a particular set of supports, torque on the parcel is not a function of orientation. This is very important for determining stable orientations.

5 Design of Velocity Fields

The motion of the parcel as it rests on a single set of supports is simply described by a set of 9 constants with mass-spring-damper qualities. A typical problem is to create a velocity field (described by $\mathbf{X}$ and $\mathbf{V}$) to produce mass-spring-damper behavior with uniform desired properties over the entire array. In particular, an equilibrium position and return spring stiffnesses are specified, and rotational equilibrium at the translational equilibrium must be ensured. The following assumptions will be made to perform the analysis:

- The coordinate origin is at the desired equilibrium.
- The cells are arranged with mirror-symmetry around the coordinate axes.
- The parcel also has mirror symmetry.
- When the parcel rotates counterclockwise, more cells under the parcel lie in the first and third quadrants than in the second and fourth. More specifically, $\sum x_i y_i > 0$.

5.1 Translational Constants

5.1.1 The Design Problem

With a viscous friction model, the translational motion of the parcel is described by

$$\vec{f} = \mathbf{k_s} \vec{X}_{cm} + \vec{f}_o - \mu W \dot{\vec{X}}_{cm} \quad (24)$$

This expression consists of 6 constants with which we can specify the dynamics of the motion of the parcel: 4 spring constants in $\mathbf{k_s}$ and two constant offset forces in $\vec{f}_o$. Consideration will only be given to the case where $\mathbf{k_s}$ is a diagonal matrix, where x and y motions of the parcel are decoupled. The diagonal terms in $\mathbf{k_s}$ tend to pull the parcel towards a central equilibrium. The off-diagonal terms act as circulatory terms, moving the parcel around the equilibrium, and are not helpful for positioning the parcel (see Figure 10). Eliminating the off-diagonal terms simplifies the design problem and improves the rotational properties of the field.

To move a parcel to a particular position, the equilibrium position $\vec{X}_{cm_e}$ and return spring strengths $k_{s_{xx}}$

simple			circulatory		
↘	↓	↙	↗	→	↘
→		←	↑		↓
↗	↑	↖	↖	←	↙

Figure 10: *A simple field (left) with diagonal* $\mathbf{k_s}$ *and a circulatory field (right) with off-diagonal entries of* $\mathbf{k_s}$.

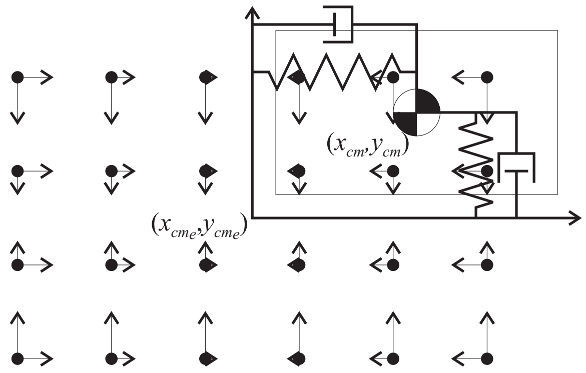

Figure 11: *With* $k_{s_{xy}} = k_{s_{yx}} = 0$ *the parcel behaves like a mass-spring damper system.*

and $k_{s_{yy}}$ will be specified. Physically, this is like the mass-spring-damper system shown in Figure 11. At equilibrium, $\vec{f_o} = -\mathbf{k_s}\vec{X}_{cm_e}$, so, in effect, $\vec{f_o}$ is specified. The design problem then becomes the problem of determining wheel velocities $\mathbf{V}$ given their positions (specified in $\mathbf{X}$ and equivalently in $\mathbf{B}$) and the constants $\mathbf{k_s}$ and $\vec{f_o}$. The section on dynamics defined the functional relationship from $\mathbf{V}$ to $\mathbf{k_s}$ and $\vec{f_o}$. In this section, a method to determine a suitable velocity matrix $\mathbf{V}$ from $\mathbf{k_s}$ and $\vec{f_o}$ is derived.

5.1.2 Restating Dynamics

The elements in $\mathbf{k_s}$ and $\vec{f_o}$ can be expressed directly from Equation 18.

$$\begin{bmatrix} f_{o_x} & k_{s_{xx}} & k_{s_{xy}} \\ f_{o_y} & k_{s_{yx}} & k_{s_{yy}} \end{bmatrix} = \mu W \mathbf{V}\mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}. \quad (25)$$

This equation can be rewritten in terms of the vector (of length $2n$) formed by stacking the transposes of the two rows of $\mathbf{V}$ ($\vec{V}_x^{\mathrm{T}}$ and $\vec{V}_y^{\mathrm{T}}$). Taking advantage of the symmetry of $\mathbf{B}\mathbf{B}^{\mathrm{T}}$, the following relation holds.

$$\begin{bmatrix} f_{o_x} \\ k_{s_{xx}} \\ k_{s_{xy}} \\ f_{o_y} \\ k_{s_{yx}} \\ k_{s_{yy}} \end{bmatrix} = \mu W \underbrace{\left[\begin{array}{c|c} \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{B} & \mathbf{0} \\ \hline \mathbf{0} & \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{B} \end{array}\right]}_{\boldsymbol{\mathcal{B}}} \begin{bmatrix} \vec{V}_x^{\mathrm{T}} \\ \hline \vec{V}_y^{\mathrm{T}} \end{bmatrix} \quad (26)$$

This produces a set of 6 equations and $2n$ unknowns.

5.1.3 Solving for Wheel Speeds

To solve for the wheel speeds ($\vec{V}_x$ and $\vec{V}_y$) it is necessary to invert the previous set of equations. Since this set of equations is underconstrained, there is some freedom in the solution and further constraints are required. The *Penrose pseudo-inverse* accomplishes this by minimizing the sum of the squares of the wheel speeds. For the non-square matrix, $\boldsymbol{\mathcal{B}}$, the Penrose pseudo-inverse, $\boldsymbol{\mathcal{B}}^{\dagger}$ is defined below, and after applying properties of matrix transposes and inverses, $\boldsymbol{\mathcal{B}}^{\dagger}$ becomes

$$\boldsymbol{\mathcal{B}}^{\dagger} \equiv \boldsymbol{\mathcal{B}}^{\mathrm{T}}\left(\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{B}}^{\mathrm{T}}\right)^{-1} = \left[\begin{array}{c|c} \mathbf{B}^{\mathrm{T}} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{B}^{\mathrm{T}} \end{array}\right]. \quad (27)$$

Therefore, given a set of constants determined by the desired equilibrium and spring constants (with $k_{s_{xy}} = k_{s_{yx}} = 0$ to decouple the x and y motions of the parcel), it is very easy to solve for the set of wheel speeds which will give a parcel the desired dynamics.

$$\begin{bmatrix} \vec{V}_x^{\mathrm{T}} \\ \hline \vec{V}_y^{\mathrm{T}} \end{bmatrix} = \frac{1}{\mu W} \left[\begin{array}{c|c} \mathbf{B}^{\mathrm{T}} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{B}^{\mathrm{T}} \end{array}\right] \begin{bmatrix} f_{o_x} \\ k_{s_{xx}} \\ 0 \\ \hline f_{o_y} \\ 0 \\ k_{s_{yy}} \end{bmatrix} \quad (28)$$

Since each row of $\mathbf{B}^{\mathrm{T}}$ contains the vector $\left[1 \; x_i \; y_i\right]$, the wheel speeds for each cell can be computed independently as

$$\begin{aligned} V_{x_i} &= f_{o_x} + k_{s_{xx}} x_i, \text{ and} \\ V_{y_i} &= f_{o_y} + k_{s_{yy}} y_i. \end{aligned} \quad (29)$$

In the diagonal $\mathbf{k_s}$ case, $f_{o_x} = -x_{cm_e} k_{s_{xx}}$, and $f_{o_y} = -y_{cm_e} k_{s_{yy}}$, which can be substituted in the previous equation, yielding

$$\begin{aligned} V_{x_i} &= k_{s_{xx}} \left(x_i - x_{cm_e}\right), \text{ and} \qquad (30) \\ V_{y_i} &= k_{s_{yy}} \left(y_i - y_{cm_e}\right), \qquad (31) \end{aligned}$$

which is a field where the cells point towards the equilibrium (for negative $k_{s_{xx}}$ and $k_{s_{yy}}$), with velocities of each component proportional to the perpendicular distance to the corresponding axis. Note that this is a discretized version of an elliptic field as described in the continuous medium case by Kavraki [5].

5.2 Rotational Constants

5.2.1 Orienting on a Discrete Field

While a parcel rests on a single set of supports, the torque is not a function of orientation. Therefore, it is not possible to construct a static velocity field which will orient a parcel more precisely than its range of motion which keeps it on a single set of supports. This free range of motion is demonstrated by the black rectangle in Figure 12. All that can be assured locally is that the parcel will be in rotational equilibrium when it is in translational equilibrium. Parcels are oriented as they change support from one cell to the next. There are then three considerations for the parcel's orientation: (i) Torque is zero when translational force is zero (at $\vec{X}_{cm} = \vec{X}_{cm_e}$). (ii) When the parcel rotates about its equilibrium position, a change in supports induces a restoring torque. (iii) Given any starting position and equilibrium, the parcel will eventually reach the desired position and orientation. These considerations will be examined under the field generated in the previous section. Even though the previous field was designed without considering orientation, it can be made to orient the parcel as well as position it.

5.2.2 Torque Equilibrium

Without loss of generality, the origin of the coordinate system will be chosen to be at the desired equilibrium position. This sets the terms of $\vec{f_o}$ to zero and simplifies the problem. When the parcel is in translational equilibrium, $\mathbf{k_s}\vec{X}_{cm} + \vec{f_o} = 0$. Also, with our choice of coordinate system, $\vec{X}_{cm} = 0$ at translational equilibrium. Therefore, when the parcel rotates about its translational equilibrium, Equation 23 reduces to

$$\tau = \tau_o - \omega\mu\vec{\mathcal{X}}\vec{N} \qquad (32)$$

Which is a constant applied torque with linear damping. For the parcel to be in complete translational and rotational equilibrium, we must then have $\tau_o = 0$. τ_o is determined from the velocity field which was designed to provide the desired translational dynamics. This constant (from Equation 23) can be written as

$$\tau_o = \mu W \left[1\ 0\ 0\right] \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \mathbf{B}\vec{R}^{\mathrm{T}}. \qquad (33)$$

The vector $\vec{R}^{\mathrm{T}}$ can be expressed in terms of the stacked velocity vector, giving

$$\begin{aligned} \tau_o = {} & \mu W \left[1\ 0\ 0\right] \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \mathbf{B}* \\ & \left[\begin{array}{ccc|ccc} -y_1 & & & x_1 & & \\ & \ddots & & & \ddots & \\ & & -y_n & & & x_n \end{array}\right] \begin{bmatrix} \vec{V}_x^{\mathrm{T}} \\ \vec{V}_y^{\mathrm{T}} \end{bmatrix}. \qquad (34) \end{aligned}$$

Now, the previous velocity field can be substituted in (with $\vec{f_o} = 0$ due to the choice of coordinate system, and with diagonal $\mathbf{k_s}$).

$$\begin{aligned} \tau_o = {} & \mu W \left[1\ 0\ 0\right] \left(\mathbf{B}\mathbf{B}^{\mathrm{T}}\right)^{-1} \mathbf{B}* \\ & \left[\begin{array}{ccc|ccc} -y_1 & & & x_1 & & \\ & \ddots & & & \ddots & \\ & & -y_n & & & x_n \end{array}\right] \left[\begin{array}{c|c} \mathbf{B}^{\mathrm{T}} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{B}^{\mathrm{T}} \end{array}\right] \begin{bmatrix} 0 \\ k_{s_{xx}} \\ 0 \\ 0 \\ 0 \\ k_{s_{yy}} \end{bmatrix} \qquad (35) \end{aligned}$$

The rows of $\mathbf{B}$ are the ones vector, the vector of x positions, and the vector of y positions. Therefore, the terms produced by multiplying $\mathbf{B}$ with other vectors and matrices form sums of the x and y components of all the cell locations. For example,

$$\mathbf{B}\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum {x_i}^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum {y_i}^2 \end{bmatrix}. \qquad (36)$$

Realizing this, Equation 35 can be rewritten in terms of these sums. After some algebra,

$$\tau_o = \mu W \left[1\ 0\ 0\right] * \begin{bmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i{}^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum y_i{}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum x_i y_i \\ \sum x_i{}^2 y_i \\ \sum x_i y_i{}^2 \end{bmatrix} (k_{s_{yy}} - k_{s_{xx}}) \quad (37)$$

In general, the torque resulting from these constants evaluated at the equilibrium position is not zero. However, consider the case where the cells on which a parcel rests are arranged *symmetrically* (mirrored in x and y) about the coordinate axes. Therefore, any term with odd powers of x_i or y_i in a summation will be identically zero. For example, in $\sum x_i y_i$, cells in the first and fourth quadrants cancel cells in the second and third quadrants, making $\sum x_i y_i{=}0$. Similarly, $\sum x_i{}^2 y_i{=}0$ and $\sum x_i y_i{}^2{=}0$. Therefore, Equation 37 reduces to

$$\tau_o = 0 \quad (38)$$

such that the parcel will be in rotational equilibrium when resting on a mirror-symmetric set of supports at the translational equilibrium.

5.2.3 Rotation About Equilibrium

The symmetric arrangement of cells under the parcel depends on the parcel's shape and orientation. Figure 12 shows a rectangular parcel in three orientations at its equilibrium position with arrows at each cell indicating the magnitudes of the velocities at each cell. In this figure, we can see that the symmetrically oriented parcel (solid black line) has a symmetric set of forces, and does not feel a torque. Similarly, the slightly perturbed parcel (dashed black line) also has a symmetric set of forces and does not feel a torque. However, the parcel which has rotated enough to change supports (dotted black line) has a set of supports which is symmetric about the origin (radially symmetric) rather than the coordinate axes and will feel a torque.

In general, Equation 37 shows that for a given arrangement of cells and a particular orientation of parcel, the torque will be either positive or negative depending on the relative magnitude of $k_{s_{yy}}$ and $k_{s_{xx}}$. Therefore, equilibrium is either stable or unstable (within cell resolution) depending on the values of these

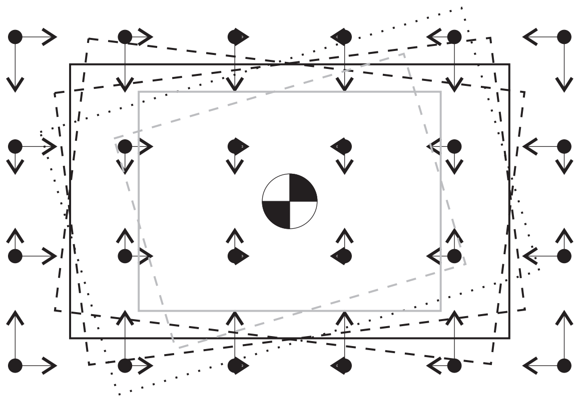

Figure 12: *Rotation of two parcels (black and grey) about their equilibrium positions. Black dashed lines represent the free (unorientable) range of motion of the black parcel. The grey parcel does not satisfy the "positive rotation" property. Note that $k_{s_{xx}}, k_{s_{yy}} < 0$ and $(k_{s_{yy}} - k_{s_{xx}}) < 0$.*

constants. If $k_{s_{xx}} = k_{s_{yy}}$ there will be no restoring torque when the parcel rotates enough to shift cells.

If the parcel is mirror-symmetric itself, a stronger statement can be made about the direction of rotation. The resulting set of supports will be radially symmetric such that for every cell (x_i, y_i) there is a cell $(-x_i, -y_i)$. Thus, many of the terms in Equation 37 become zero.

$$\begin{aligned} \tau_o &= \mu W \left[1\ 0\ 0\right] * \\ &\quad \begin{bmatrix} n & 0 & 0 \\ 0 & \sum x_i{}^2 & \sum x_i y_i \\ 0 & \sum x_i y_i & \sum y_i{}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum x_i y_i \\ 0 \\ 0 \end{bmatrix} (k_{s_{yy}} - k_{s_{xx}}) \\ &= \frac{1}{n} \sum x_i y_i \left(k_{s_{yy}} - k_{s_{xx}}\right). \end{aligned} \quad (39)$$

A final assumption can be made. When the symmetric parcel rotates counterclockwise about the equilibrium position, $\sum x_i y_i > 0$. This is often true, since more of the parcel is in the first and third quadrants, and more first and third quadrant cells (giving $x_i y_i > 0$) are covered. The black rectangle in Figure 12 shows a parcel with this property. However, because of the discreteness of the array, some parcels (for example, the grey rectangle in Figure 12) may have a negative $\sum x_i y_i$ for some counterclockwise rotations. A particular parcel can be checked for "orientability" on the array by analyzing it as it rotates about equilibrium and to see which cells it covers. For parcels with this "positive rotation" property, there will be a restoring torque for $\left(k_{s_{yy}} - k_{s_{xx}}\right) < 0$ with a stable orientation.

5.2.4 Arbitrary Starting Locations

The previous rotational analysis examined a symmetric parcel whose center of mass is at its equilibrium position, and the cells are arranged symmetrically (such as in a regular array) about this equilibrium. Nothing was said for torques on the parcel when the position is not at the equilibrium. In order to assure proper orientation, the parcel must first reach its equilibrium and then orient itself.

Reaching the equilibrium is guaranteed regardless of orientation and cell distribution. Since the parcel's dynamics are that of a mass-spring-damper centered at the origin independent of which cells are supporting it, the parcel will eventually settle to its positional equilibrium. When the orientation changes, changing the supports, the translational dynamics are unaffected.

Once position equilibrium is reached, a torque will be applied which tends to orient the parcel. For $\left(k_{s_{yy}} - k_{s_{xx}}\right) < 0$, a symmetric parcel will rotate until it is aligned with the coordinate axes. This will happen as long as more first and third quadrant cells are covered when the parcel rotates counterclockwise.

6 Conclusions and Future Work

In this paper, a simplified computation of the dynamics of a parcel was done for the MDMS array. This analysis involved computing the distribution of the weight of a flat parcel over an arbitrary discrete set of flexible supports. The interaction between the constant-speed wheels and the parcel was modeled using a viscous friction model. A matrix formulation was used to sum up the forces on the parcel, and the resulting motion of the parcel is that of a mass-spring-damper, with spring, offset, and damping constant over a particular set of supports.

A standard wheel velocity field was methodically designed to provide uniform, arbitrary spring constants and equilibrium position over the entire array regardless of which cells support the parcel. A pseudo-inverse was used to solve this underconstrained problem which provides the desired behavior while minimizing wheel speed energy. The resulting field, when only the non-circulatory spring constants were used, was an inward-pointing field with each wheel's velocity proportional to its the corresponding component of perpendicular distance to the equilibrium position.

The derived field provides uniform, smooth motion for the parcel even though the supporting forces and points of actuation change discontinuously. This is because the moment equilibrium of the parcel uses the positions of the cells as moment arms, just as the velocities in the field vary linearly with cell position. When the parcel changes supports, the normal forces change in proportion with the wheel velocities providing a continuous horizontal force.

This field was then analyzed for its rotational equilibrium properties. Interestingly, the torque on the parcel is not a function of the orientation because the supporting forces only depend on the position of the center of mass. Therefore, the transitions from cell to cell as the parcel rotates about its equilibrium are used to orient the parcel within the resolution limits of the discrete array. Given the assumption that the set of cells supporting the parcel at equilibrium is mirror-symmetric about the coordinate axes, it was found that the parcel will be in rotational equilibrium at the translational equilibrium. A ramification of this assumption on a regular square-lattice array, such as the MDMS, is that the equilibrium position must be set either exactly on a cell or exactly midway between cells. Also, a parcel can be oriented only to angles of $\frac{n\pi}{4}$, for $n = 0, \ldots, 3$.

The torque developed by the inward-pointing field was analyzed as the parcel rotates about equilibrium. It was determined that, as long as the two spring constants are not equal, when the parcel rotates and the supports change, there will be a restoring torque on a symmetric parcel with the property that $\sum x_i y_i > 0$ for all the cells under the parcel when the parcel rotates counterclockwise. This positive rotation property is an artifact of the discrete array and holds for many parcel sizes and shapes. Furthermore, in the the limit of many cells spaced closely together (e.g., a continuous array), this property always holds for symmetric parcels, as implied by Kavraki. Finally, a simple analysis can be done to check if a parcel meets this assumption.

Physical intuition can be applied to explain the restoring torque. For example, consider a cell in the first quadrant with coordinates (x, y). This cell will apply a force in the direction $(k_{s_{xx}}x, k_{s_{yy}}y)$ (where $k_{s_{xx}}, k_{s_{yy}} < 0$.) The moment about the origin of this force will be $-k_{s_{xx}}xy + k_{s_{yy}}xy = (k_{s_{yy}} - k_{s_{xx}})xy$. If $(k_{s_{yy}} - k_{s_{xx}}) < 0$, first quadrant cells apply a negative torque, as will third quadrant cells, while second and fourth quadrant cells apply a positive torque. If the parcel rests on a mirror-symmetric set of cells, these torques will cancel. If the parcel rotates, there will be a restoring torque if the first and third quadrant cells outweigh the second and fourth quadrant cells, or, if $\sum x_i y_i > 0$.

When the approach described in this paper is applied to generate a specific vector field that has a single equilibrium position and orientation (to symmetry), this method results in a discretized version of the elliptic potential fields examined by Kavraki. However, the general framework here considers the distribution of weight among supports, while at the same time models the interaction between parcel and actuators, and accounts for the discreteness of the array. This is an extension of the work done by Kavraki and others like Böhringer, who do not need to consider these issues because of the scale of their tasks.

Many of the assumptions, such as symmetry, made in this paper are not necessary and may be relaxed in the future. The positive rotation property will be examined, and a more definite statement about parcel orientability will be made. Another issue for future research is the relationship between the discretized field and the continuous field. Many intuitive ideas operate on both fields, but it is difficult to analyze one in terms of the other. A more unified approach to handle both types of fields is desirable.

In general, there are limitations to a passive (open-loop) policy such as this, particularly on a discrete array where positioning and orientation can only be done within the resolution of the array. An active policy which updates wheel velocities base on feedback from the motions of the parcel would be necessary to more precisely manipulate parcels. Also, the policies in this paper rely on a full-slip contact between the wheels and parcel. A full-stick contact would be better suited to precise manipulation with an active policy in order to have better control over the parcel.

References

[1] S. Akella, W. Huang, K. Lynch, and M. Mason. Sensorless parts feeding with a one joint robot. In *Proceedings. Workshop on the Algorithmic Fundamentals of Robotics*, 1996.

[2] K. Böhringer, B. Donald, R. Mihailovich, and N. MacDonald. A theory of manipulation and control for microfabricated actuator arrays. In *Proceedings. IEEE International Conference on Robotics and Automation*, 1994.

[3] M. Erdmann. An exploration of nonprehensile two-palm manipulation: Planning and execution. In *Proceedings. Seventh International Symposium on Robotics Research*, 1995.

[4] K. Goldberg. Orienting polygonal parts without sensors. *Algorithmica: Special Issue on Computational Robotics*, 10:201–225, August 1993.

[5] L. Kavraki. Part orientation with programmable vector fields: Two stable equilibria for most parts. In *Proceedings. IEEE International Conference on Robotics and Automation*, 1997.

[6] J. Luntz, W. Messner, and H. Choset. Parcel manipulation and dynamics with a distributed actuator array: The virtual vehicle. In *Proceedings. IEEE International Conference on Robotics and Automation*, 1997.

[7] J. Luntz, W. Messner, and H. Choset. Virtual vehicle: parcel manipulation and dynamics with a distributed actuator array. In *Proceedings of SPIE vol. 3201. Sensors and Controls for Advanced Manufacturing. International Symposium on Intelligent Systems and Advanced Maufacturing.*, 1997.